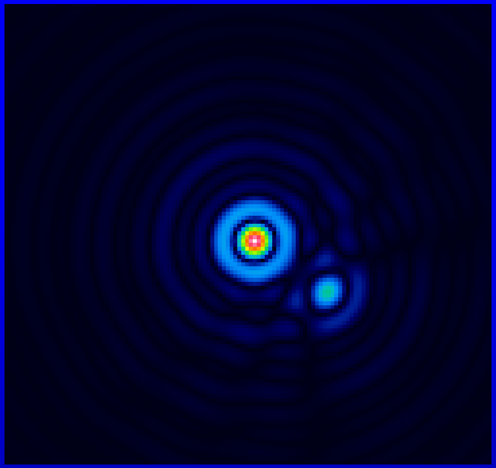
ASD and Speckle Interferometry

Dave Rowe, CTO, PlaneWave Instruments

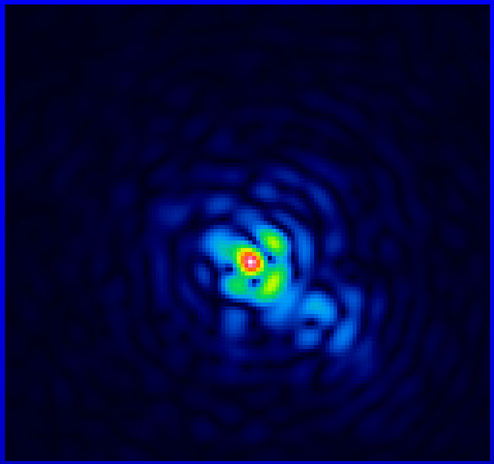
Part 1:
Modeling the
Astronomical Image

Static



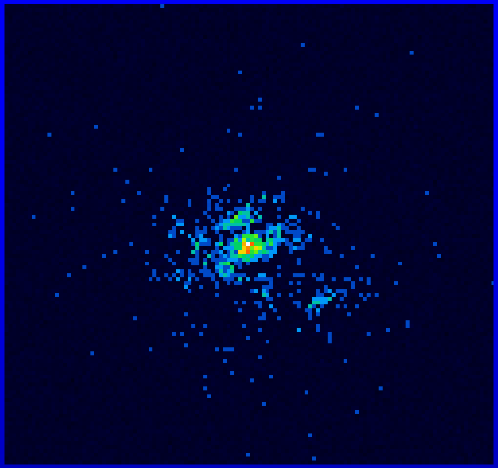
Start with Object,
add
Diffraction and
Telescope Aberrations

Dynamic



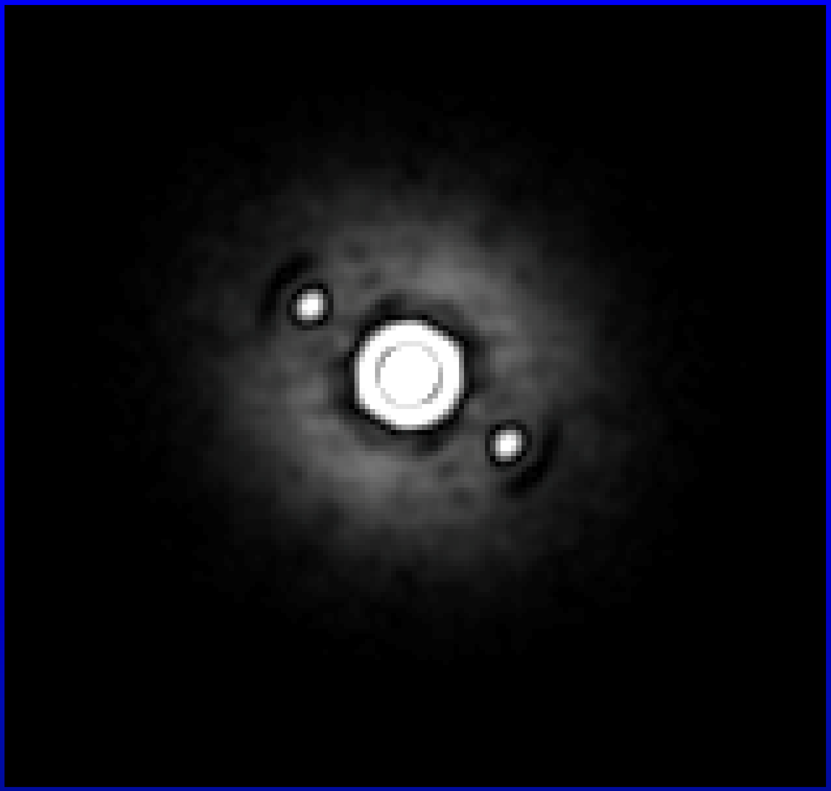
add
Atmospheric Seeing Distortion
(ASD)

Stochastic



add
Photon Statistics
and Camera Noise

Part 2:
Speckle Interferometry Reduction

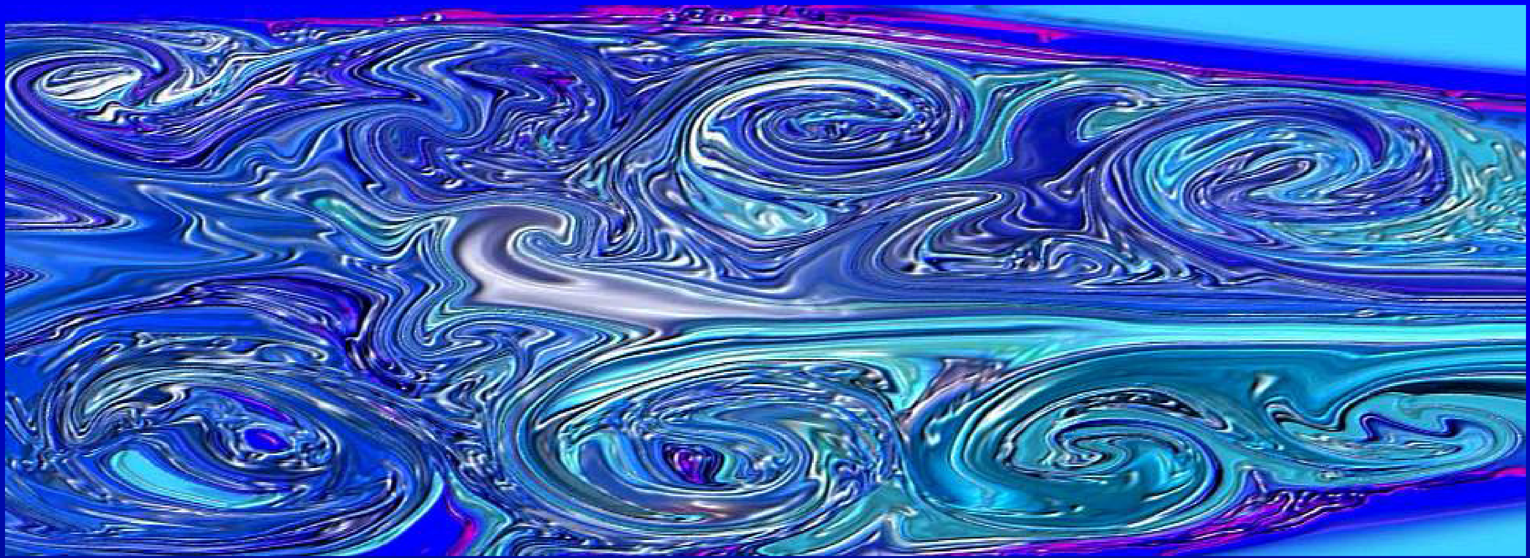


Average the Power Spectrum,
Calculate the Autocorrelation

Modeling the Atmosphere: Turbulence

Big whirls have little whirls that feed on their velocity,
and little whirls have lesser whirls and so on to viscosity.

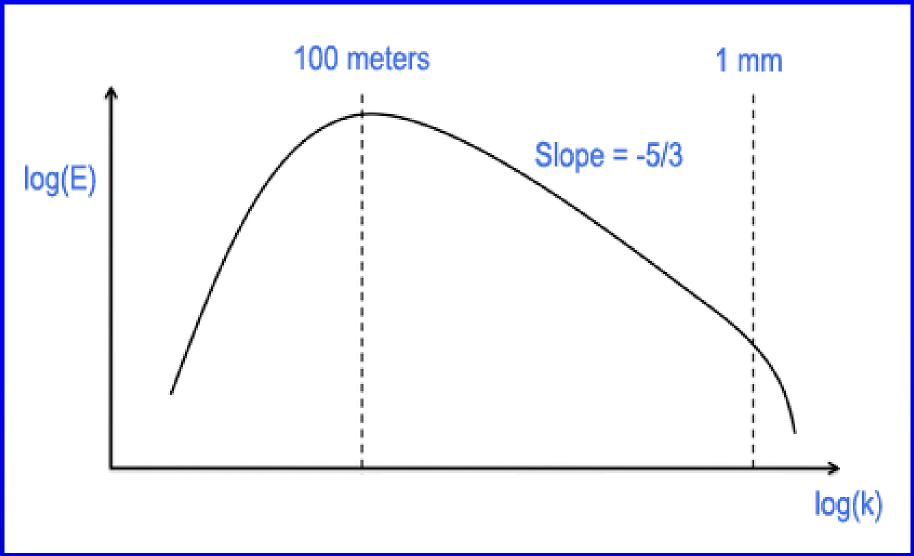
- Lewis Fry Richardson



Credit: Ashley Davidoff

Atmospheric Turbulence has significant temperature and density fluctuations from the Outer Scale (~100 meters) to the Inner Scale (~1 mm)

Kolmogorov Turbulence Spectrum



Energy spectrum of atmospheric turbulence versus spatial frequency, log-log plot

Kolmogorov length scale depends only on viscosity and rate of energy dissipation

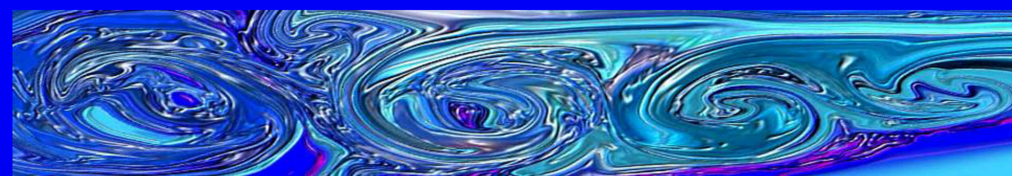
$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4}$$

Katsushika Hokusai, *The Great Wave off Kanagawa*

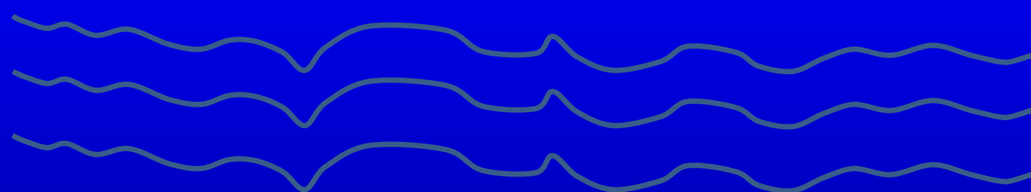
Propagation of Light through the Atmosphere

Wavefront Phase
at top of atmosphere

Temperature (density) fluctuations cause differences in the index of refraction and velocity of propagation



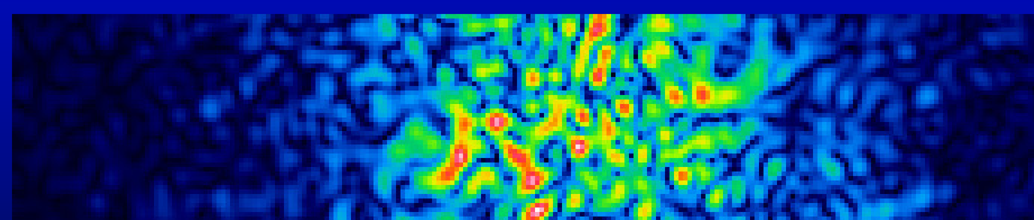
Wavefront Phase
at entrance pupil
(add aberrations, if desired)



Fourier Transform

↓

Instantaneous
Image



Modeling Atmospheric Seeing Distortion (ASD)

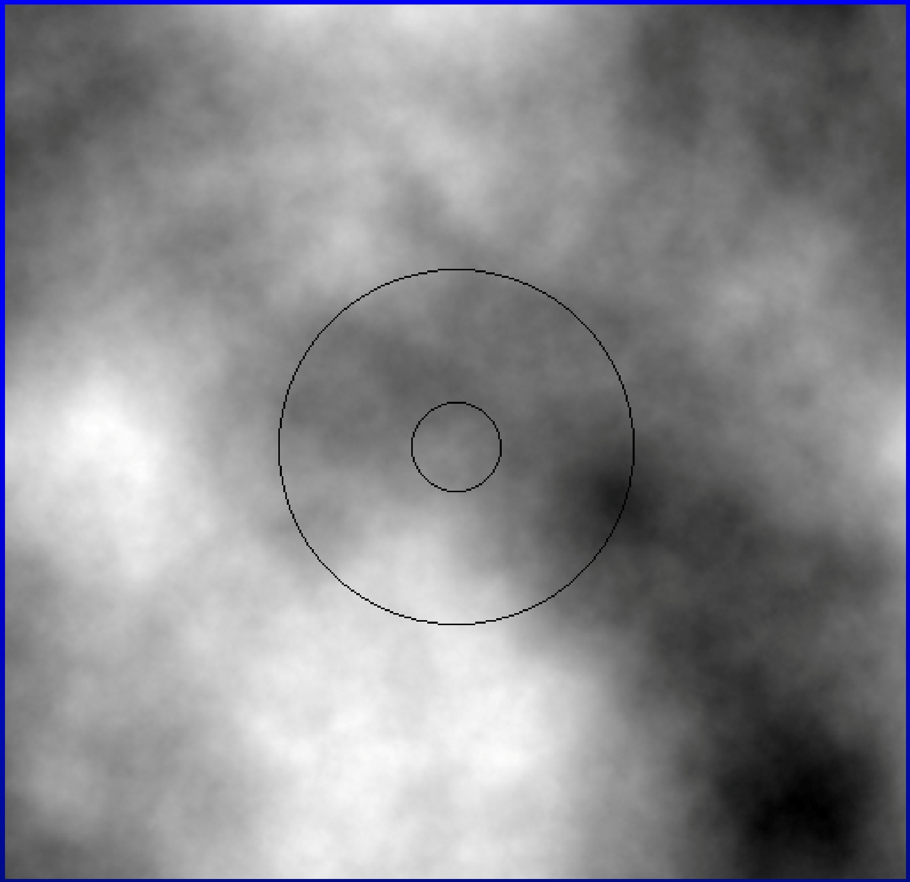
- Create 2D array of random numbers having Gaussian distribution with Mean = 0 and Sigma = 1
- Multiply this array by the modulus (amplitude) of the wavefront phase spectral density (PSD) as given by V.I. Tatarski, “Wave Propagation in a Turbulent Medium”
- Fourier Transform to generate a specific occurance of the wavefront phase

The modulus squared of the phase PSD is given by

$$\varphi(k) = \alpha {r_0}^{5/3} k^{-11/3}$$

where α is a normalization constant, r_0 is the Fried parameter, and k is the spatial frequency

Example of Simulated Phase through the Atmosphere

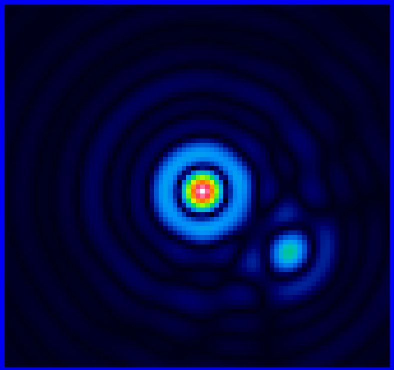


Pixel brightness represents the wavefront phase incident on the telescope's entrance pupil, either advanced or retarded

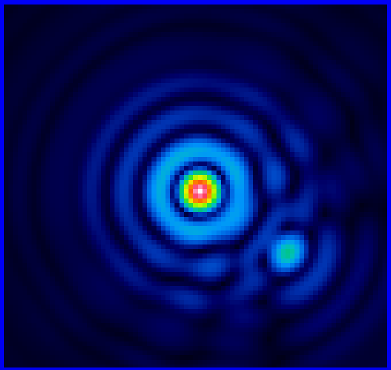
Outer circle is telescope aperture, inner circle is central obstruction

There's a good reason it looks like a cloud – the spatial frequency characteristics are identical

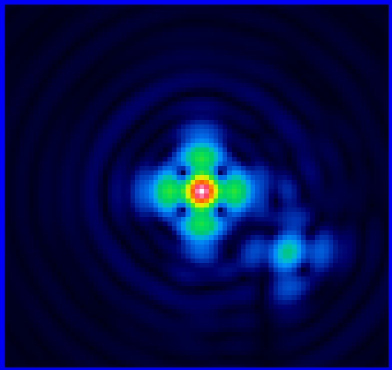
Simulated Optical Aberrations (without the atmosphere)



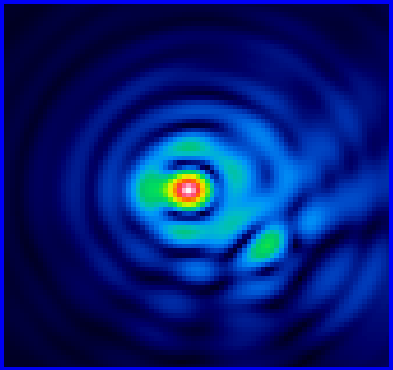
No Aberrations



½ wave PV
Spherical



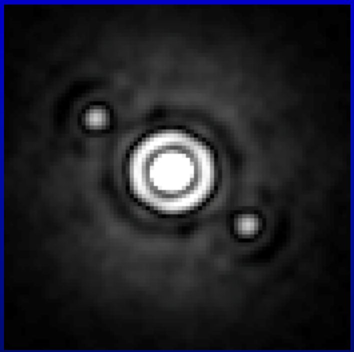
½ wave PV
Astigmatism



½ wave PV
Sph + Astig + Coma

Preview:
The Magic of
Speckle Interferometry

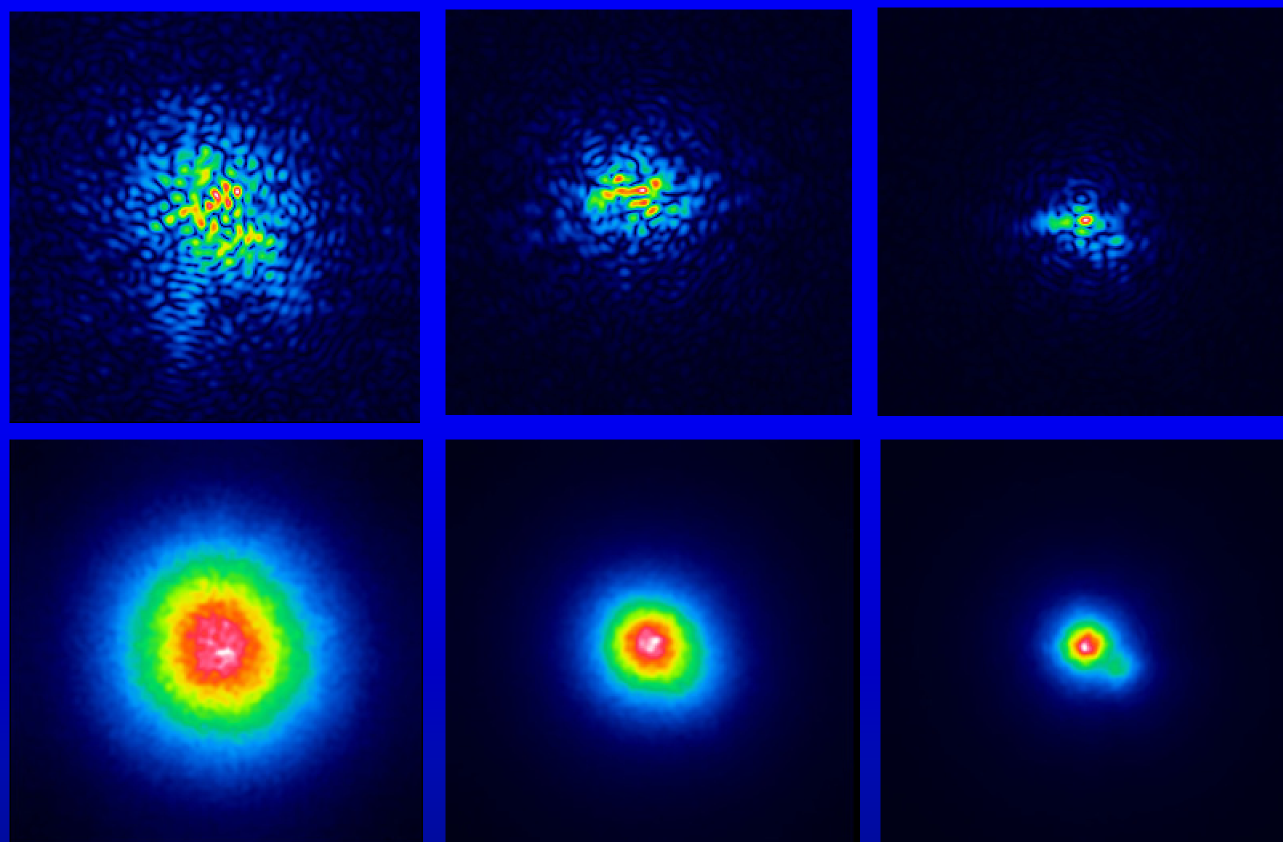
500 processed images
having
½ wave P-V
of Sph + Astig + Coma



Effect of Fried Parameter on the Image

Instantaneous PSF

Average PSF

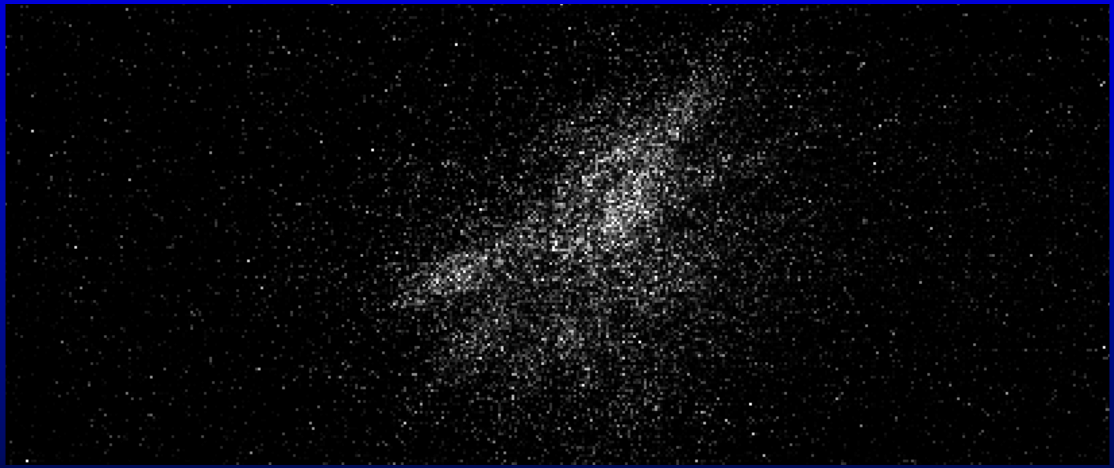


Fried Diam	5 cm	10 cm	20 cm
Ave PSF Diam	2.4” FWHM	1.3” FWHM	0.8” FWHM

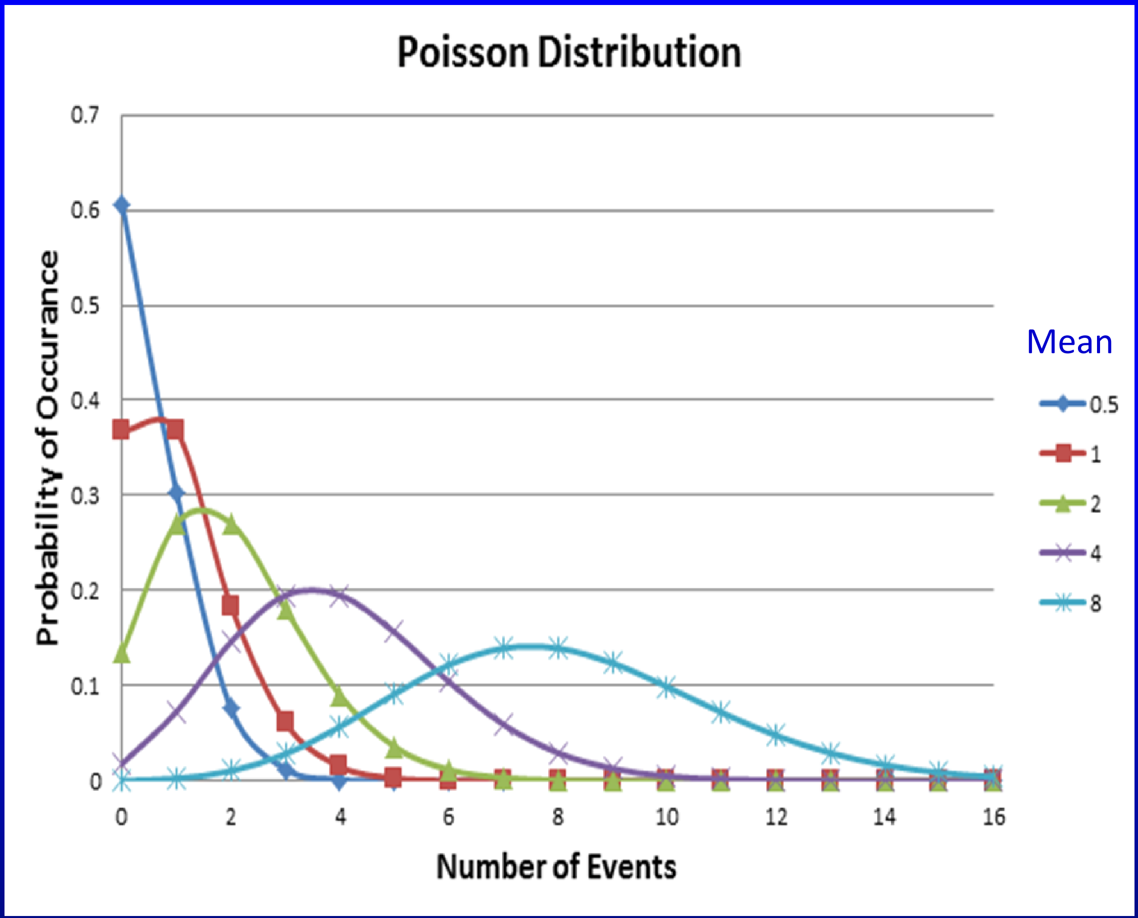
Modeling Noise

- For low brightness objects, photon shot noise dominates
- Conventional simulated image intensity is actually the photon arrival rate
- Given the arrival rate for each pixel, use Poisson statistics and a random number generator to simulate individual photons
- For EMCCD, multiply by gain and add read noise

Image from
Andor Luca-R
EMCCD Camera
taken using 2.1-meter
at Kitt Peak
WDS00101+3825
I-band , 40 mS



Poisson Statistics



- Discrete probability distribution
- Probability of a given number of events occurring in a fixed time interval
- Correctly describes the number of photons arriving at a given pixel in a given integration period when the mean number of photons is known

How many photo-electrons?

12th mag A0 star
0.5 meter telescope with CO = 40%
0.03 second Exposure
I-band with BW = 85 nm
QE = 30%, Optical Transmission = 60 %

Aperture Diam	50	cm		**for each exposure**		
Obstruction Diam	20	cm		Signal	53.8	
Elevation	300	meters MSL		Photon Shot Noise	10.4	100.0%
Air Mass	1.4			Scintillation	0.0	0.0%
Object Brightness	12	magnitude		Read Noise	0.0	0.0%
Image Scale	0.05	arcsec/pixel		Sky Glow Noise	0.1	0.0%
Gaussian PSF FWHM	0.01	arcseconds		Encircled Energy	100.00%	
Aperture Diam	1	pixels		Dark Current Noise	0.2	0.0%
Exposure	0.03	seconds		Flat-fielding Noise	0.0	0.0%
Number of Exposures	1000			Total Noise	10.4	
Read Noise	0	e-/pixel		**SNR**	**5.2**	
Dark Current	1	e-/pixel/sec			**192.7**	mMag
Sky Background	14	mag/arcsec^2				
Shot Noise Penalty	3	dB		**for all exposures**		
RMS Flat-fielding Error	0.00%			**SNR**	**164.1**	
Scintillation Coefficient	0				**6.1**	mMag

Source: CCD_SNR_V2.3.xls

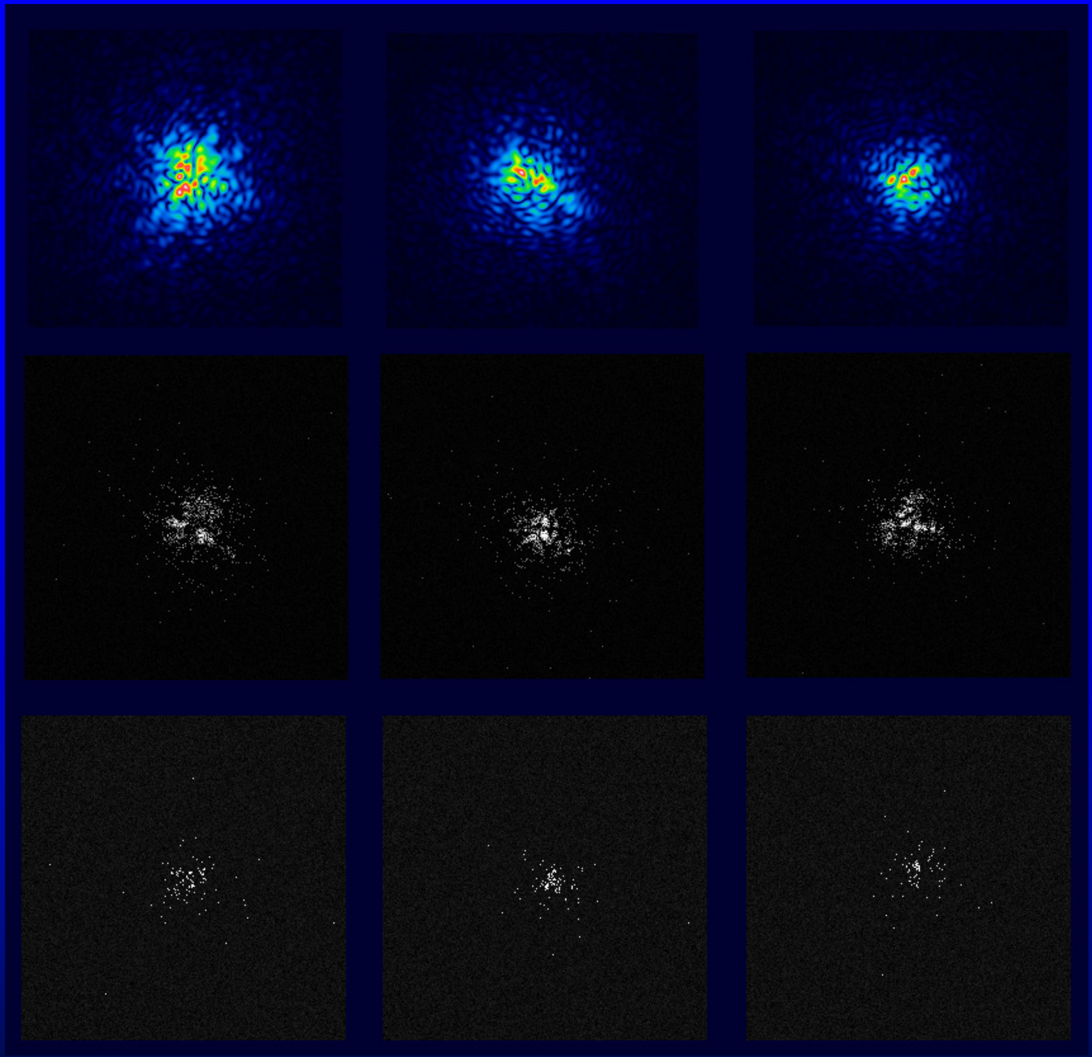
Simulated Noise Example

Fried Cell Diam. = 10cm Sep = 1” Delta Mag = 0.1   Ap Diam. = 50 cm CO = 40%

Conventional simulated image is the photon arrival rate

Poisson Statistics 1000 e- / image

Poisson Statistics 100 e- / image



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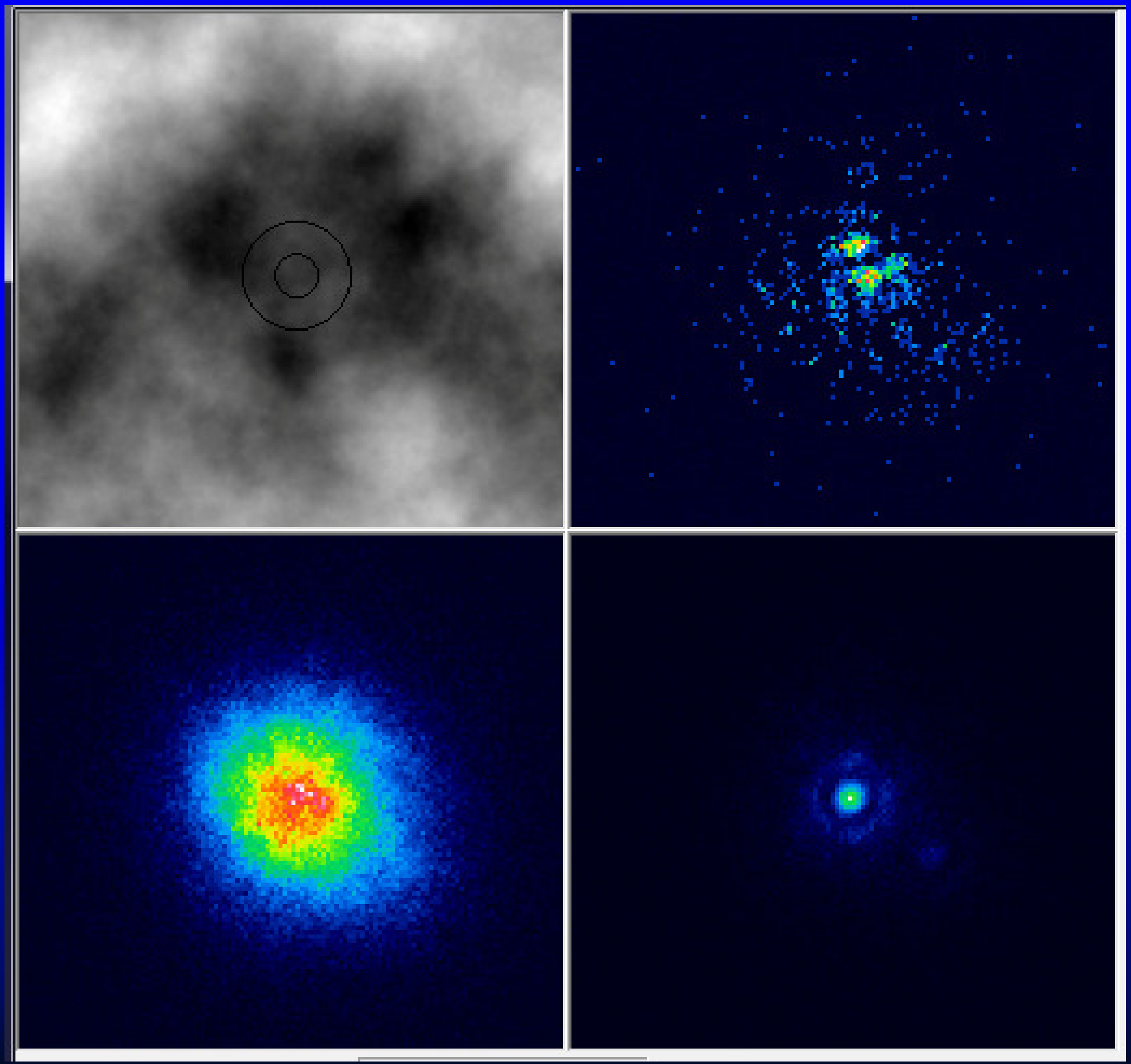
ASD Simulation Example

Fried Diam. = 10 cm   Ap Diam. = 50 cm   CO = 40%   1000 e- / image

Phase Screen -- Brightness represents wavefront phase at entrance pupil

Instantaneous Image



Time Exposure 1000 images

Lucky Image 100/1000 images

Part 2:
Speckle Interferometry

- First employed by Antoine Labeyrie (1970)
- The idea can be understood in either the image domain or the spectral domain.
- Take a large number of short exposures to freeze the seeing
- Average the power spectra
- Take the inverse Fourier transform to find the autocorrelation

Credit: IAU

Speckle Interferometry

image = object convolved with the total PSF
(aperture, aberrations, atmosphere)

$$i(\mathbf{x}) = o(\mathbf{x}) \circledast psf(\mathbf{x})$$

Fourier transform of the image is the
Fourier transform of the object times
the Optical Transfer Function

$$I(\mathbf{k}) = O(\mathbf{k}) \cdot OTF(\mathbf{k})$$

Average the modulus squared
over many images

$$< I(\mathbf{k})^2 > = O(\mathbf{k})^2 \cdot < OTF(\mathbf{k})^2 >$$

Deconvolve in the Fourier domain with
the OTF of a single star

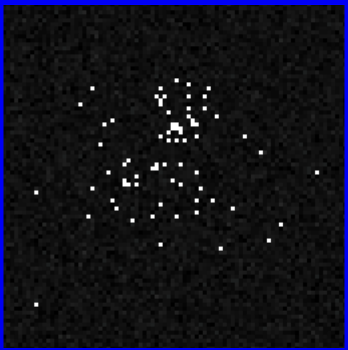
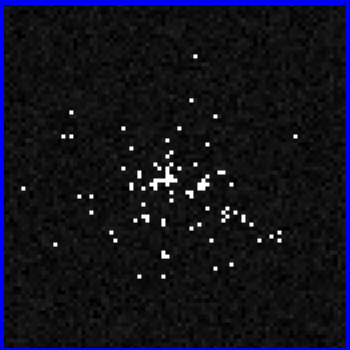
$$O(\mathbf{k})^2 = < I(\mathbf{k})^2 > / < OTF(\mathbf{k})^2 >$$

Calculate the autocorrelation from the
estimate of the modulus squared
by taking the inverse Fourier transform

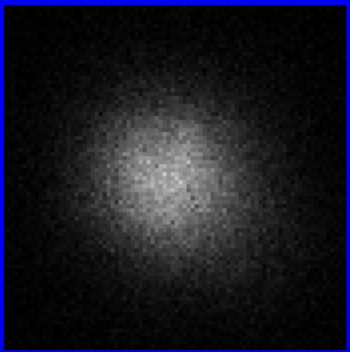
$$a(\mathbf{x}) = FT\left\{\sqrt{O(\mathbf{k})^2}\right\}$$

Simulated Example

100 e- / image, Fried Cell Size = 10 cm
Telescope: 50 cm aperture, 40% obstruction
Double Star: 1” separation, 2.5 mags diff

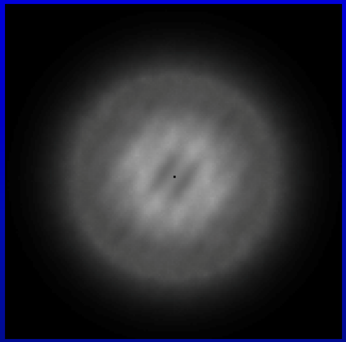




3 of 1000 images with 100 e- per image on average

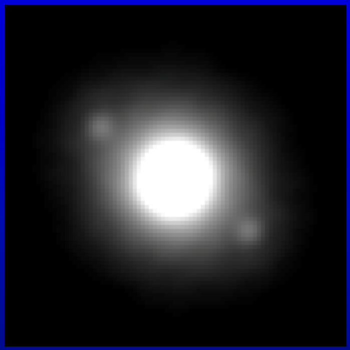


Averaged

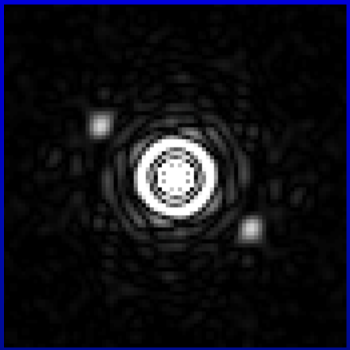
Lucky Stacked



Averaged
Power Spectrum



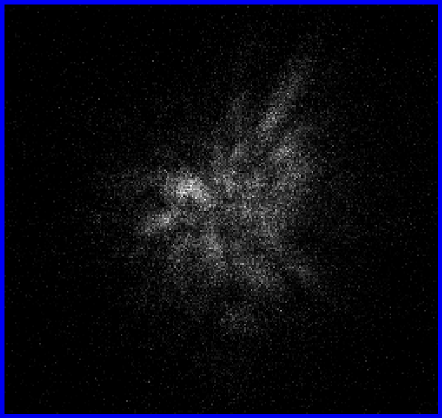
Autocorrelation



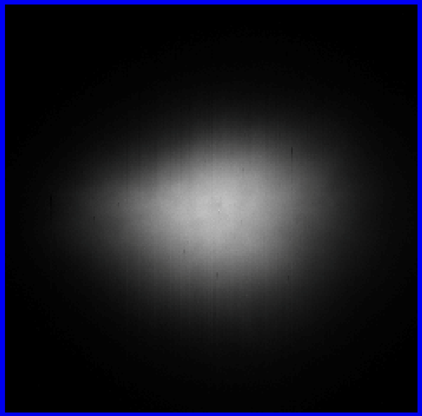
Autocorrelation
with Deconvolution

Example from Kitt Peak (Genet, 2013)
2.1 meter telescope

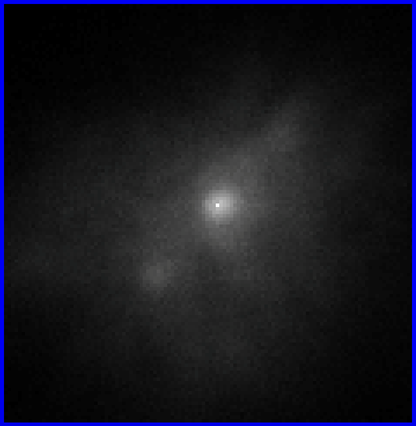
WDS00355+4006 (Brightness = 8.09, 9.73 Sep = 0.32")



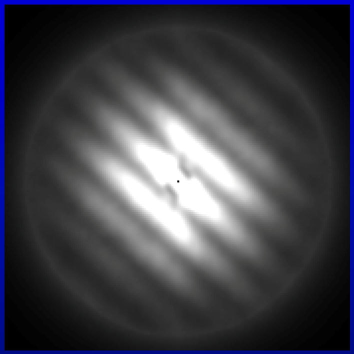
Typical Image



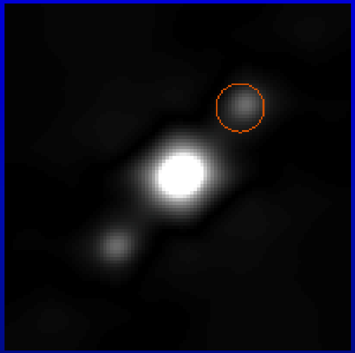
Average of 1000



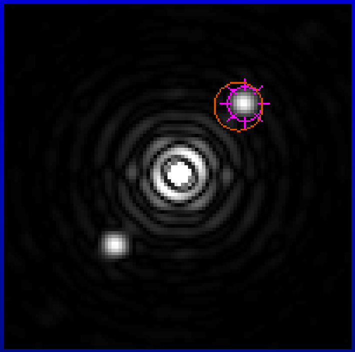
Lucky Stacked



Average
Power Spectrum



Autocorrelation
(FFT of PSD)

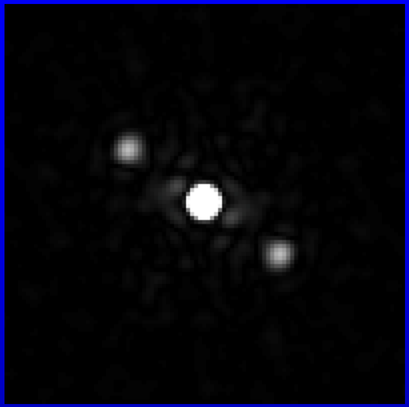


Autocorrelation
with Deconvolution

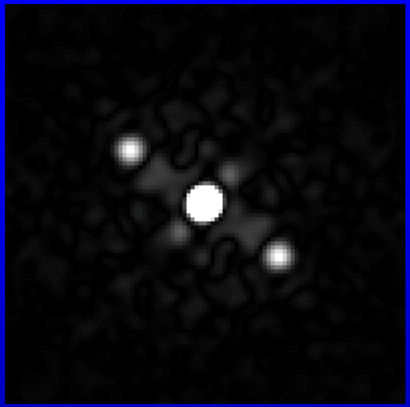
Effect of Telescope Aberrations

Autocorrelation as a function of telescope aberrations
SAC = Spherical + Astigmatism + Coma
deconvolution was used

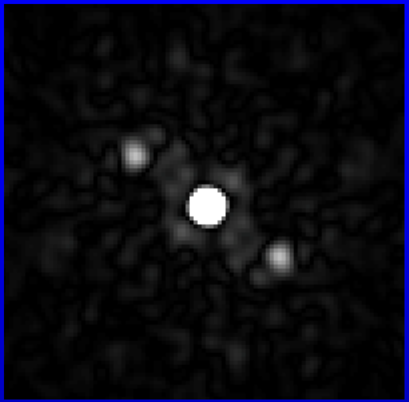
No Aberrations



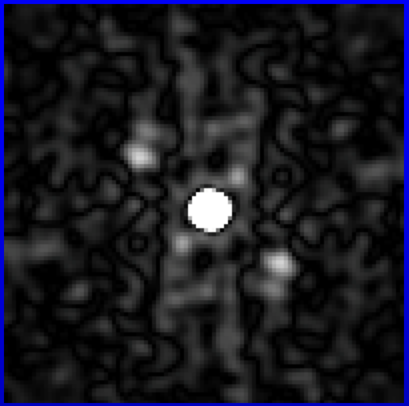
½ wave SAC



1 wave SAC



2 waves SAC

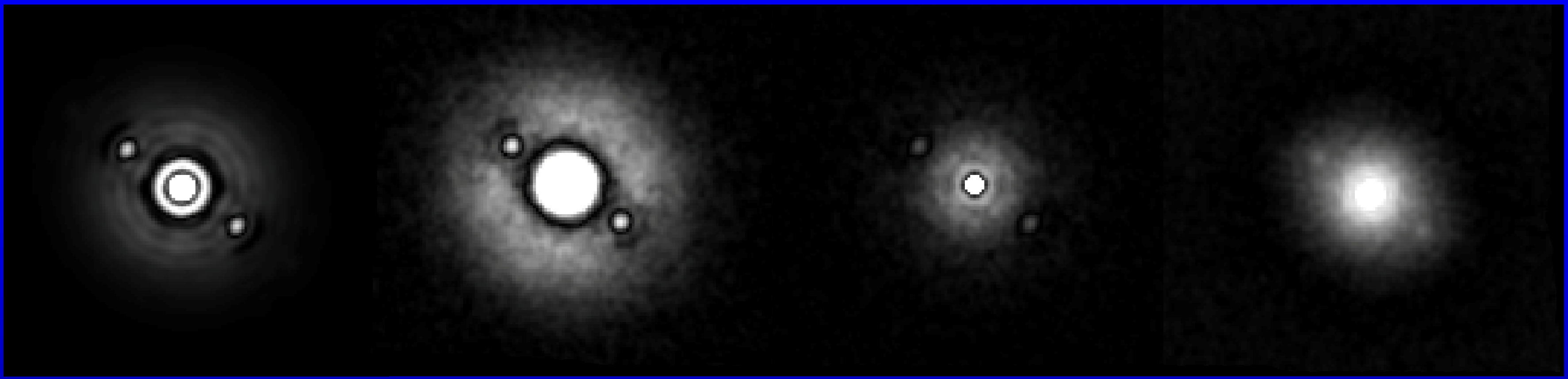


Fried Cell Diam. = 10 cm (seeing = 1.3” FWHM)
Double Star: Separation = 1”   Brightness Diff = 2.5 magnitudes
500 images
0.5-meter telescope with 40% CO, 300 e-/image

Effect of Photon Shot Noise

Autocorrelation as a function of the number
of electrons detected per image

No Shot Noise 100 e- / image 50 e- / image 25 e- / image

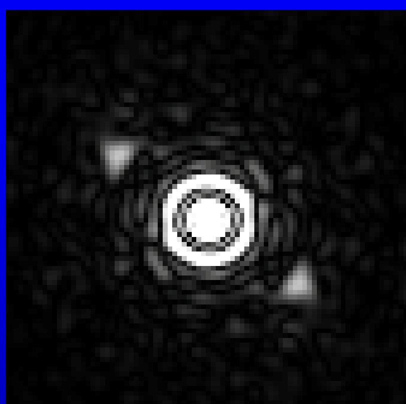


Fried Cell Diam. = 10 cm
Double Star: Separation = 1” Brightness Diff = 2.5 magnitudes
1000 images
0.5-meter telescope with 40% CO

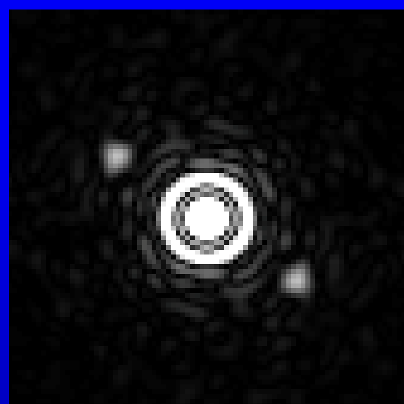
Effect of Image Averaging

Autocorrelation as a function of the number of images averaged
(deconvolution with bright star was used)

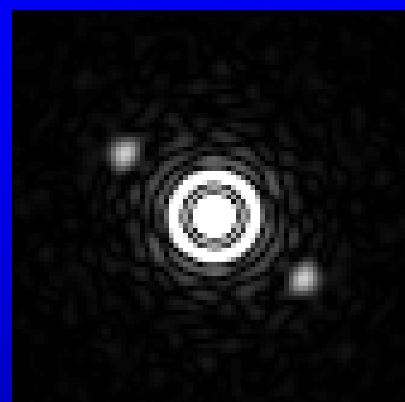
200 images



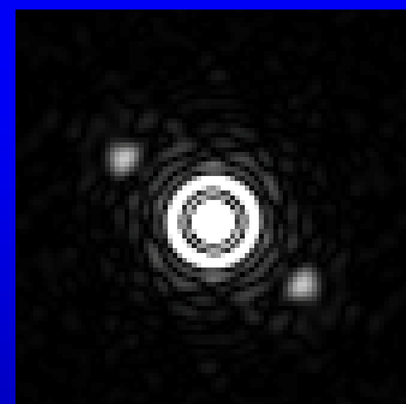
400 images



800 images



1600 images



Fried Cell Diam. = 10 cm (seeing = 1.3” FWHM)
Double Star: Separation = 1”   Brightness Diff = 2.5 magnitudes
0.5-meter telescope with 40% CO, 100 e-/image

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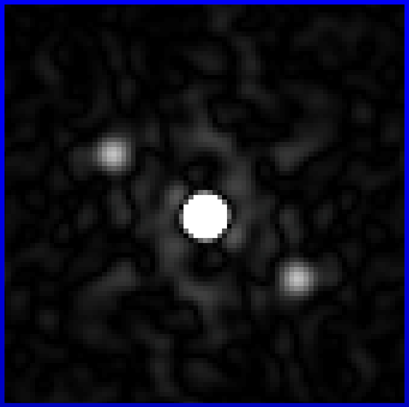
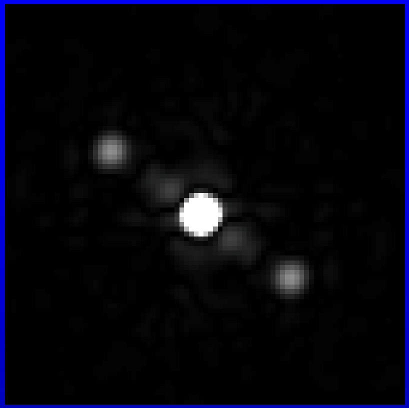
Effect of Fried Parameter

Autocorrelation as a function of the Fried cell size
in each case deconvolution was used

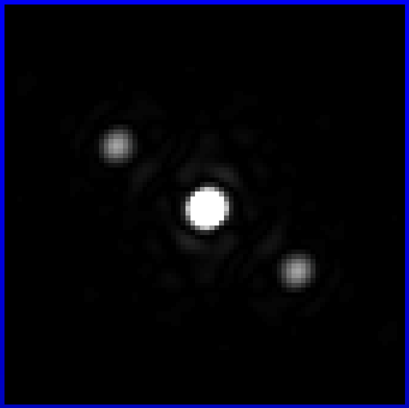
5 cm



10 cm



20 cm



Double Star: Separation = 1” Brightness Diff = 2.5 magnitudes
0.5-meter telescope with 40% CO, 300 e-/image
500 images

Summary

- Atmospheric seeing produces coherent distortion
- We can model ASD, telescope aberrations, photon statistics and camera noise
- Speckle interferometry reconstructs the image with high resolution and good SNR, but phase is lost
  - Take short exposures to freeze the seeing
  - Average the power spectra
  - Take the inverse Fourier transform to find the autocorrelation
- Future work: use bispectrum analysis to recover the phase