Basic Theory of Speckle Imaging

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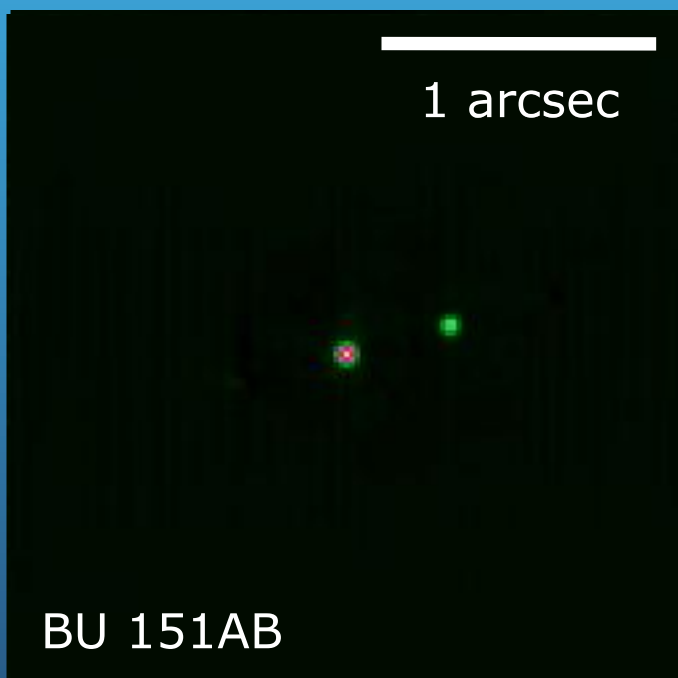
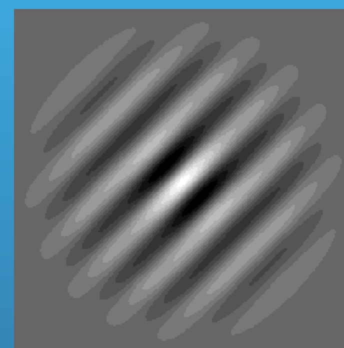
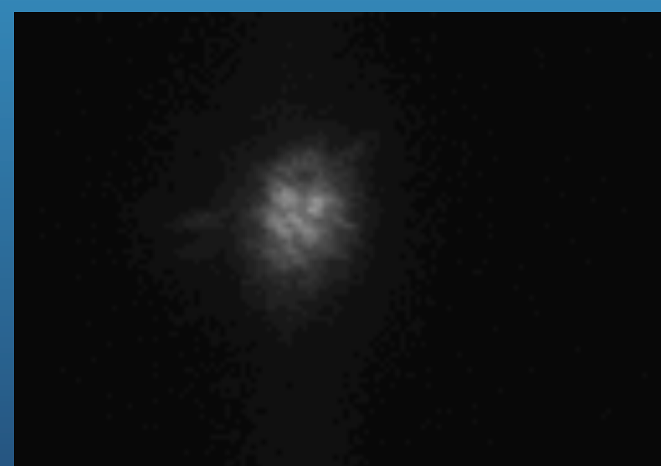






Speckle Often Means Binary Stars

- Stellar Masses.
- Mass-Luminosity Relation (MLR)
- Initial Mass Function (IMF)
- Statistics of binaries as clues to star formation and galactic evolution.
  - Ghez et al, Leinert et al. Recent models of Bate, etc.
  - Duquennoy & Mayor.
  - Post-formation environment.
- Binary Stars can host exoplanets.

Predict image quality

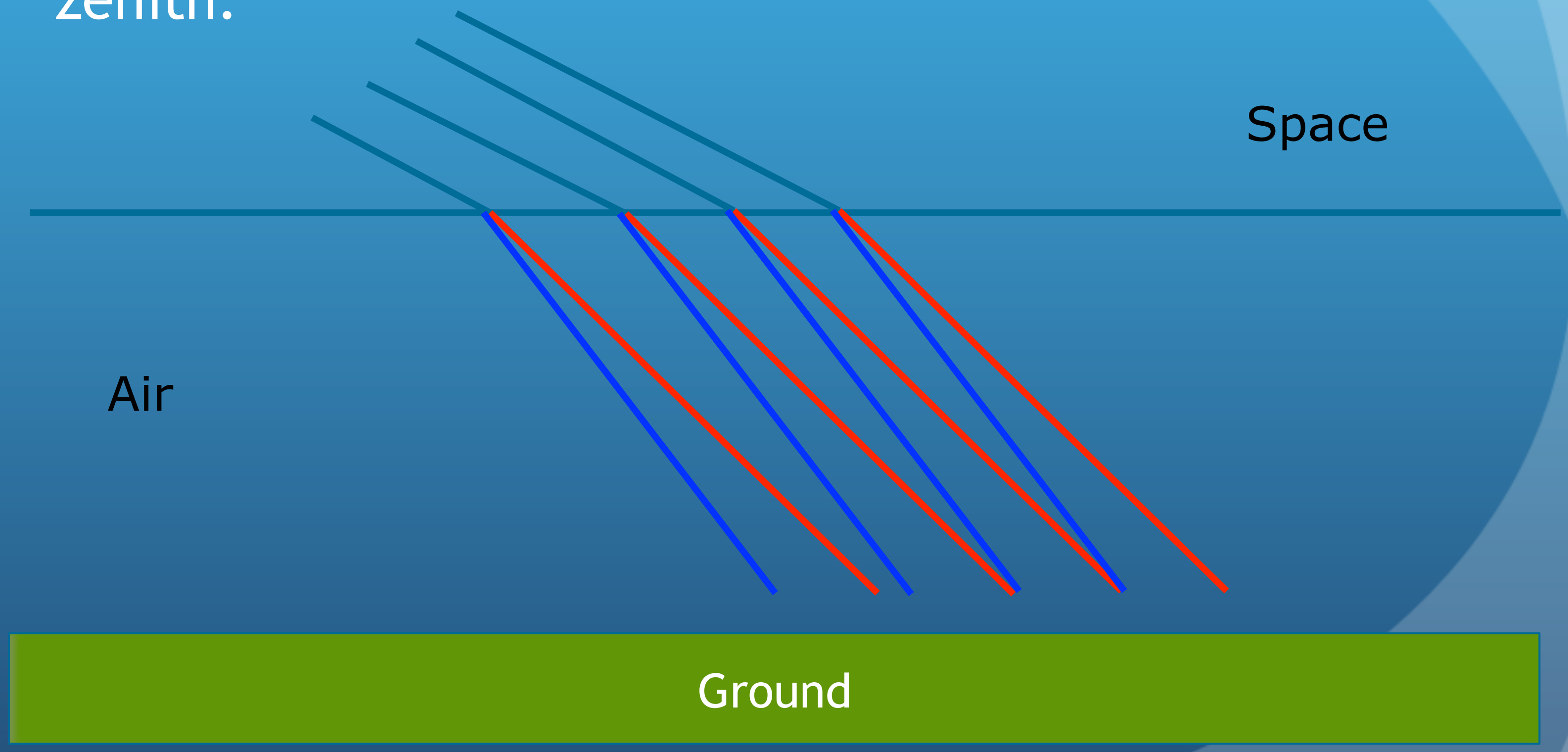
- Large Telescope with good optics:

$$\theta_R = 1.22\frac{\lambda}{D}$$

- For a wavelength of 550 nm, D=8 m, should be able to resolve objects down to 83 nanoradians, or converting to arc seconds, 0.017 arcseconds.
- But typical image quality, even at the best sites, is ~40x worse, about 0.7 arcsec.

First effect: dispersion

- Spreads light out along a direction pointing to the zenith.



Two effects

- "Fishbowl" effect: stars appear closer to zenith than they actually are. Affects position, not image quality.

- Color effect: blue light from object appears closer to the zenith than the red light. Affects image quality.

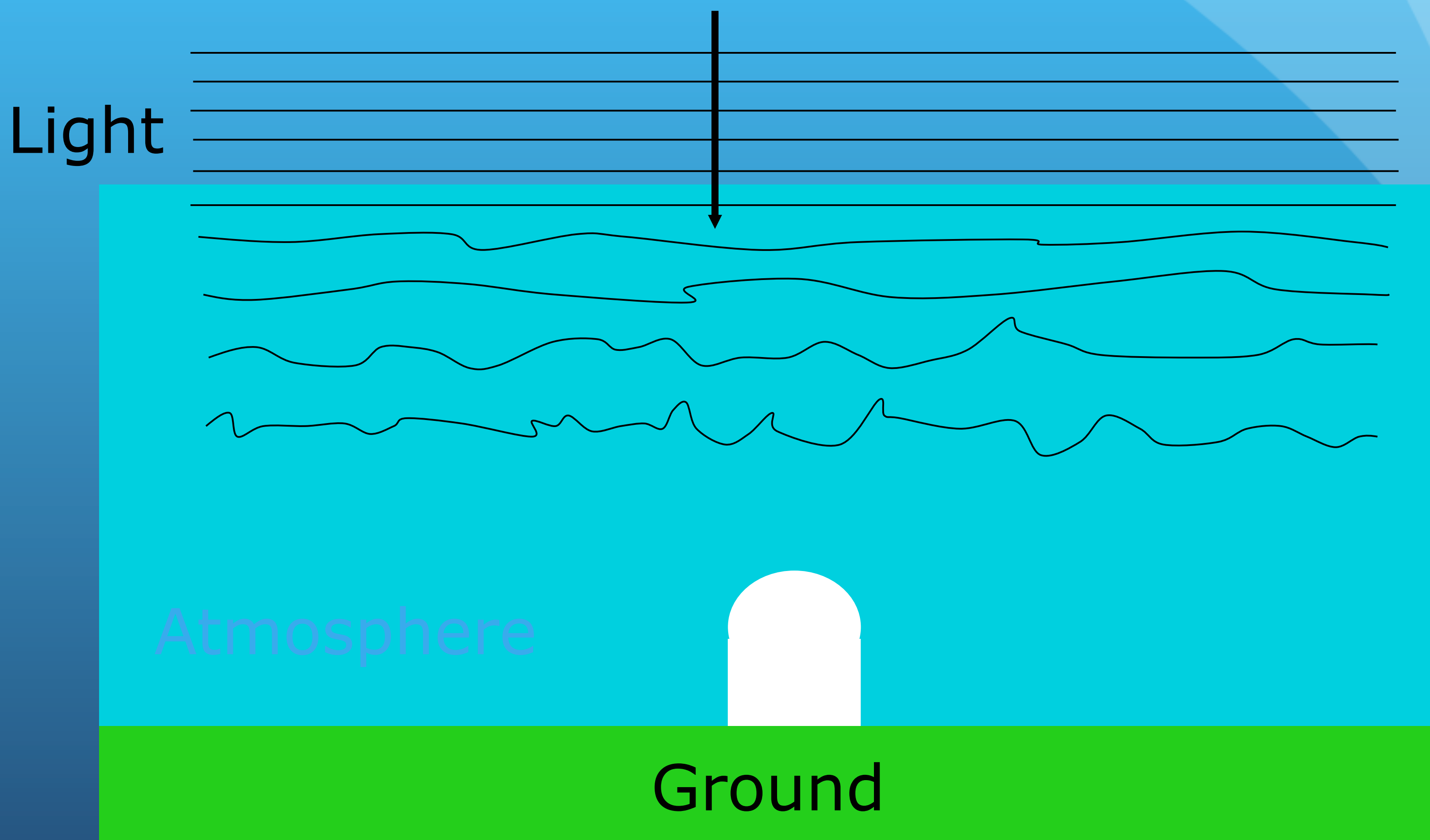
Air Turbulence

- The air is turbulent. So, temperature and pressure variations in air mean that the index of refraction of air is inhomogeneous.
  - If plane waves come down through the atmosphere, the optical path length difference varies depending on what path is taken through the air. Characteristic coherence length: 10cm.
- The magnitude of the E-field also varies due to photon statistics.
  - Example: ~10000 photons per square meter per nm per second for a 0^{th} magnitude star (Vega). Implies 1% variation in intensity just from Gaussian statistics.

Small aperture versus large aperture

- Small aperture
  - Scintillation: intensity of image varies with time.
  - Tip and tilt: Single image but it appears to wander.
  - AKA twinkling.
- Large aperture
  - Scintillation is much less noticeable (on bright objects), since the number of photons per unit time is larger.
  - Image breaks into several to many “bright points” called speckles.

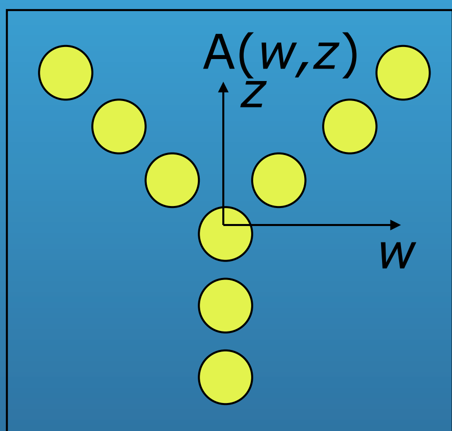
Atmospheric Turbulence



Interferometry Tutorial

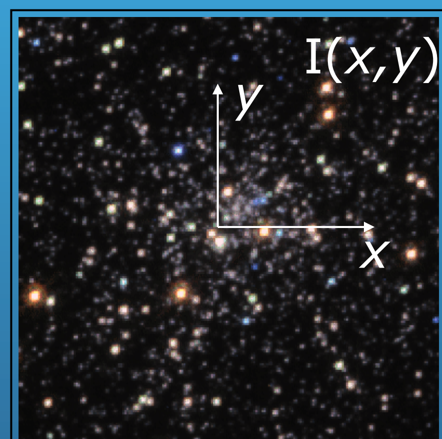
- Three “spaces.”

Aperture



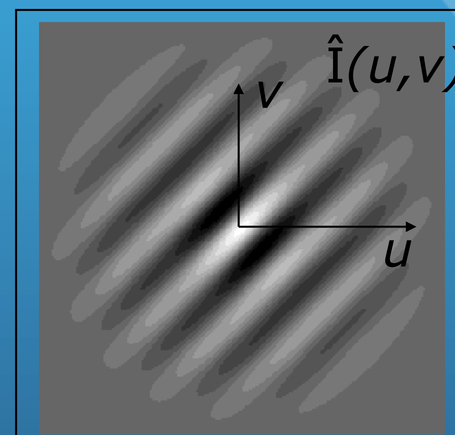
(w,z)

Image



(x,y)

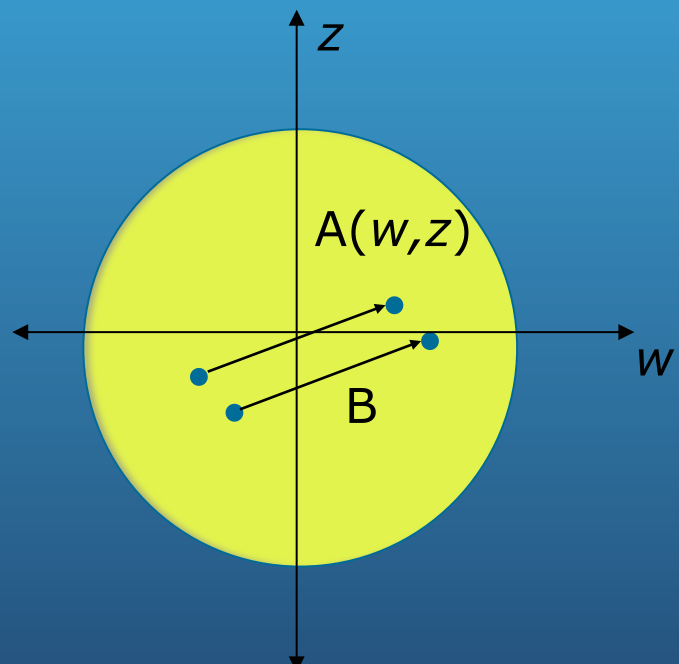
Spatial Frequency

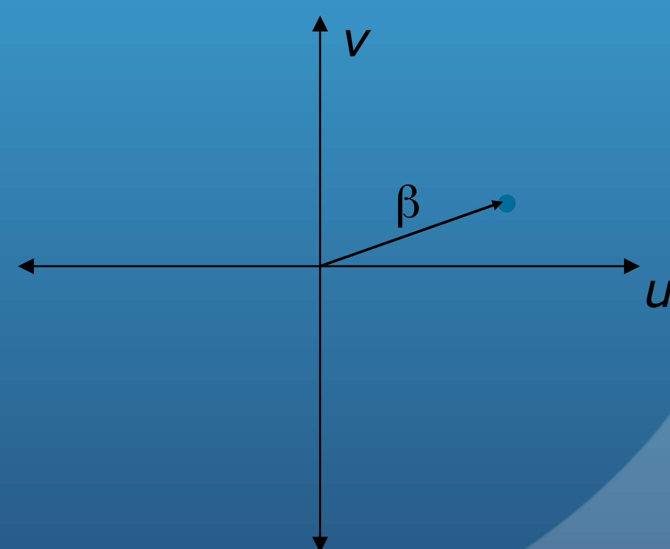


(u,v)

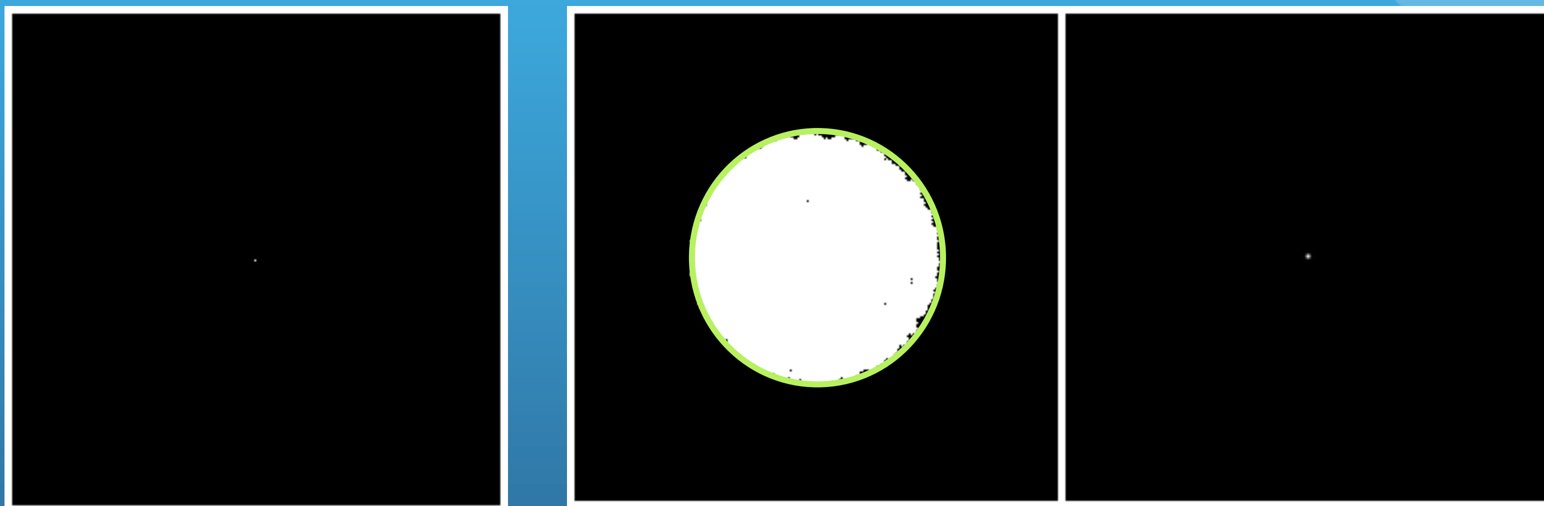
Baselines

- Define a baseline (B). That baseline contributes to 1 and only 1 Fourier component (β) of the image.
- Connects (w,z)-plane to (u,v)-plane.



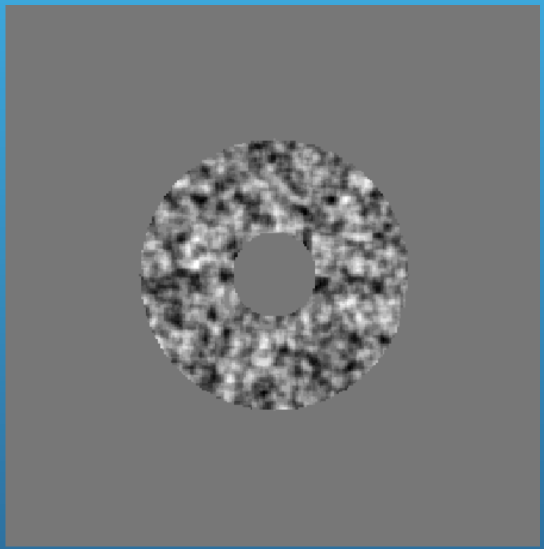


Aperture Synthesis

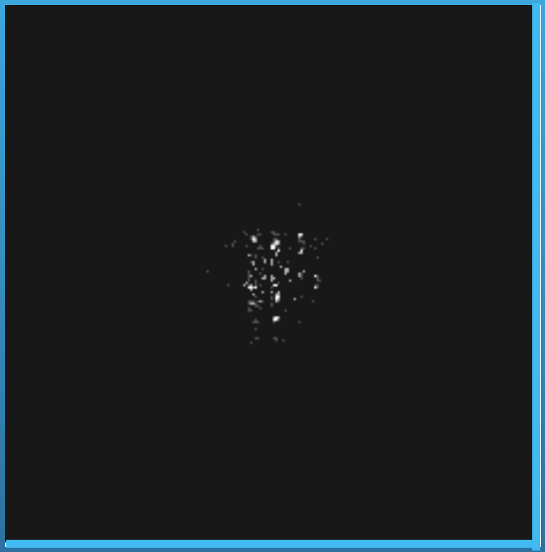


Point Source Aperture Image

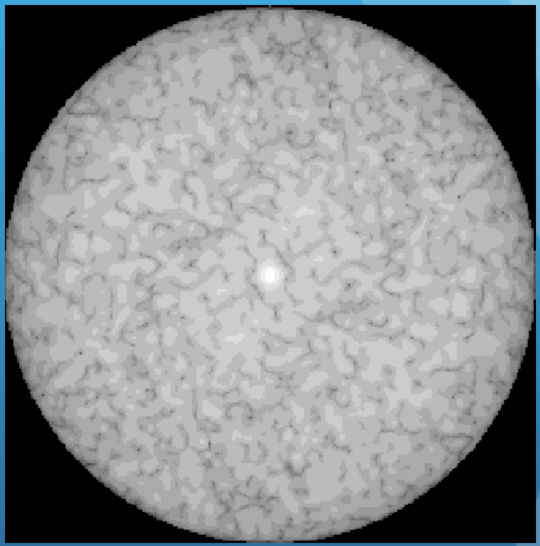
Point Source, Telescope, Atmosphere



(w,z)



(x,y)



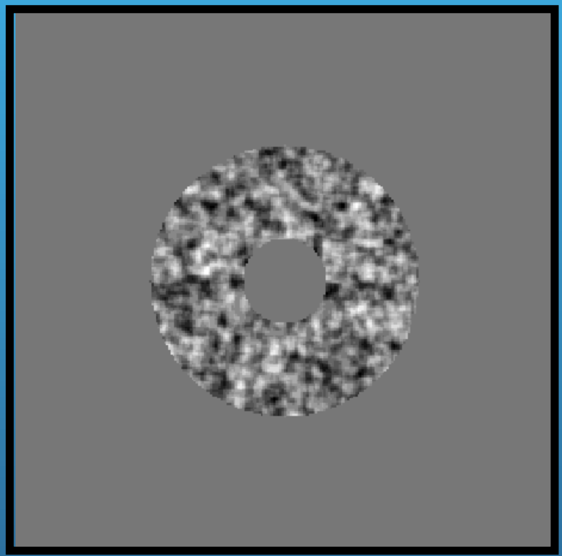
(u,v)

Spatial, Temporal Coherence

- Speckle lifetime ~10's of ms, spatial scale ~10cm.
- Long exposure images:
  - Speckles wash out, leave overall envelope of speckles.
  - Envelope size determined by spatial scale of atmosphere.
  - No high resolution information is left.
- Can we retrieve the information somehow from short exposure images?

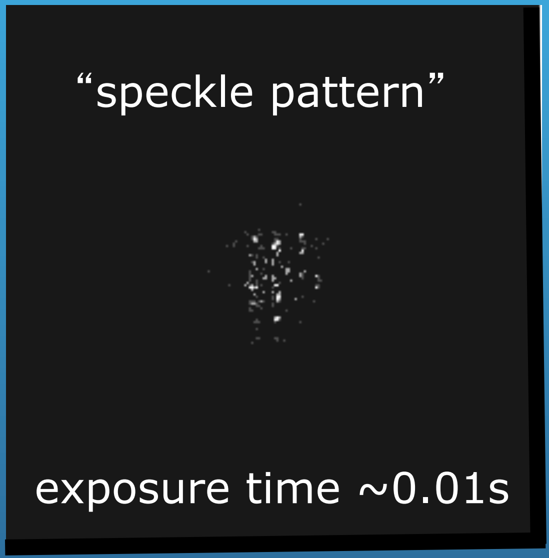
The atmosphere dictates the point spread function

Aperture



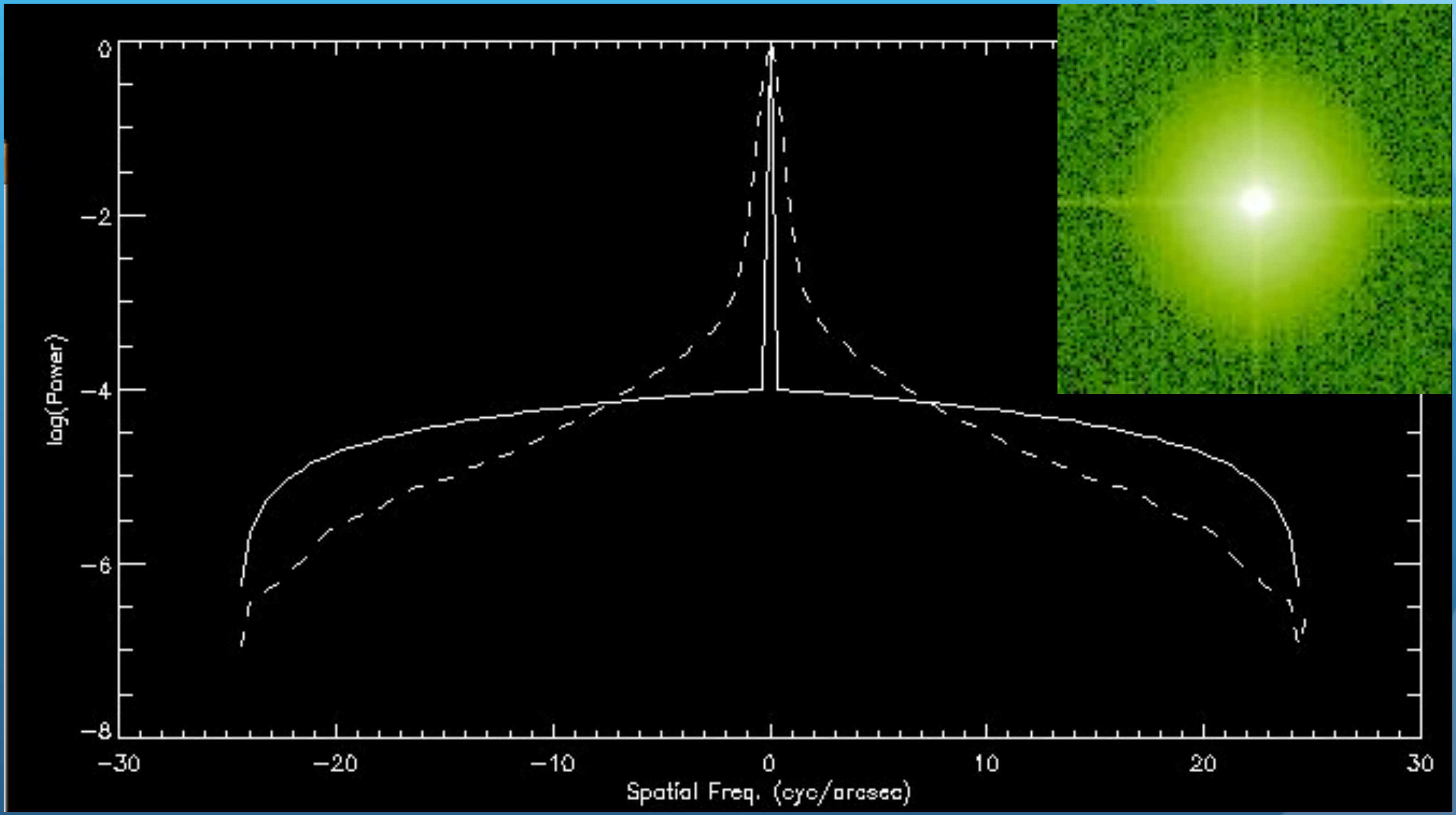
(w,z)

Image



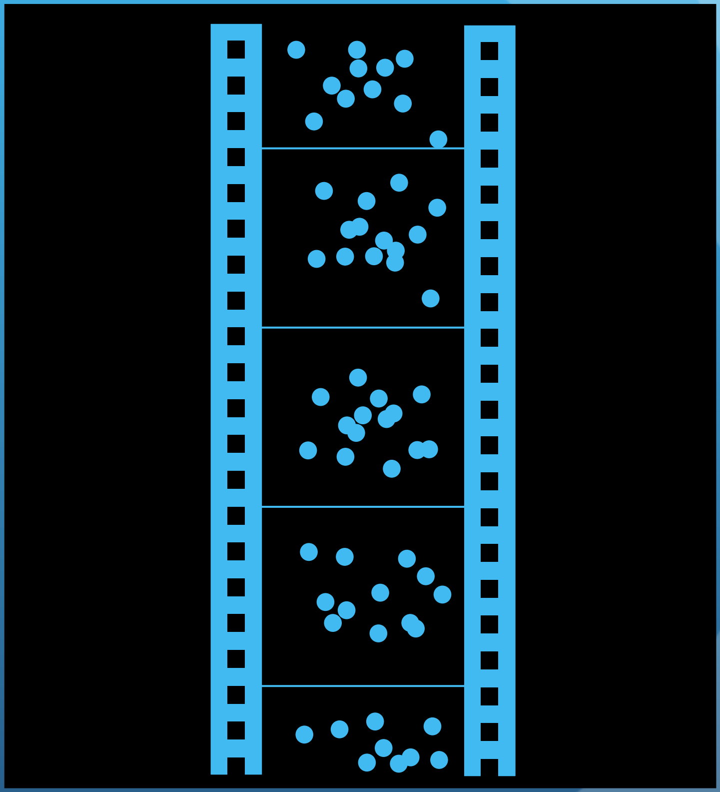
(x,y)

A Model Power Spectrum versus Reality

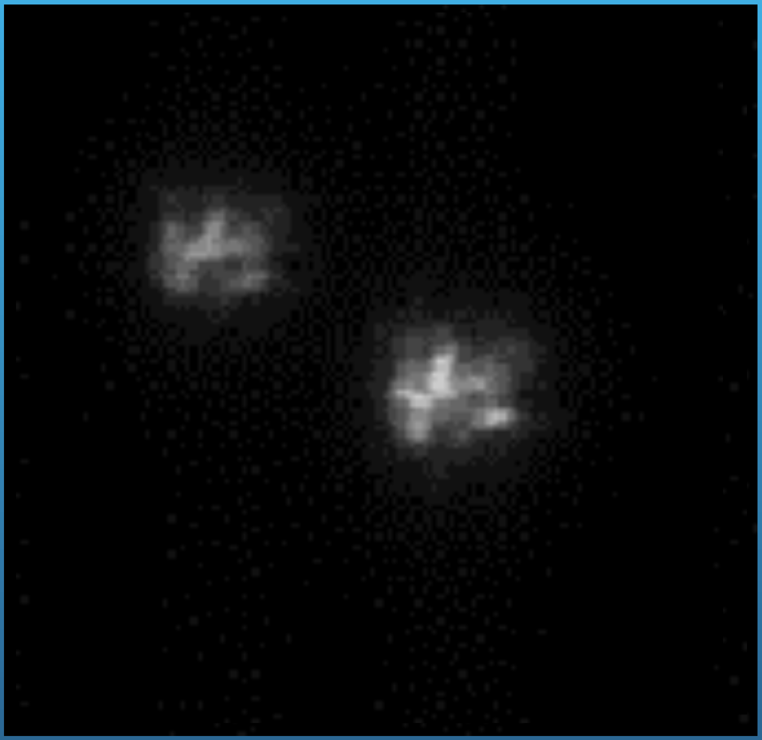


Speckle Interferometry in a Nutshell

- Make a "movie"
- Each frame is a unique speckle pattern
- Analyze data frame by frame.
- "Passive" technique



Binary Star Images



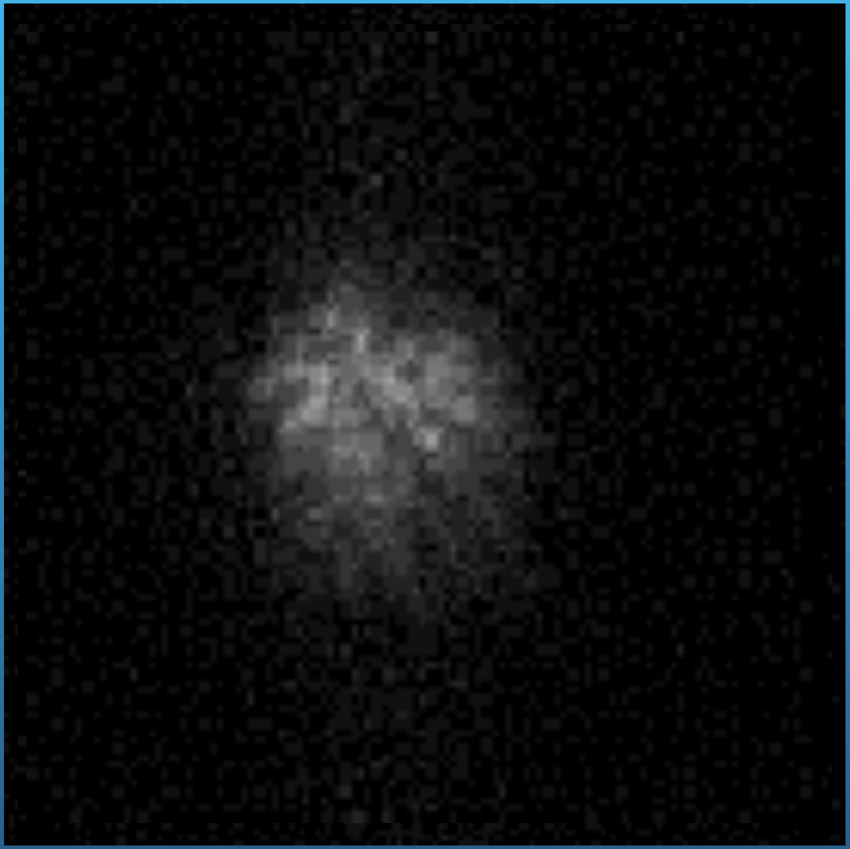
t=0.00s

t=0.05s

t=0.10s

t=0.15s

A smaller separation



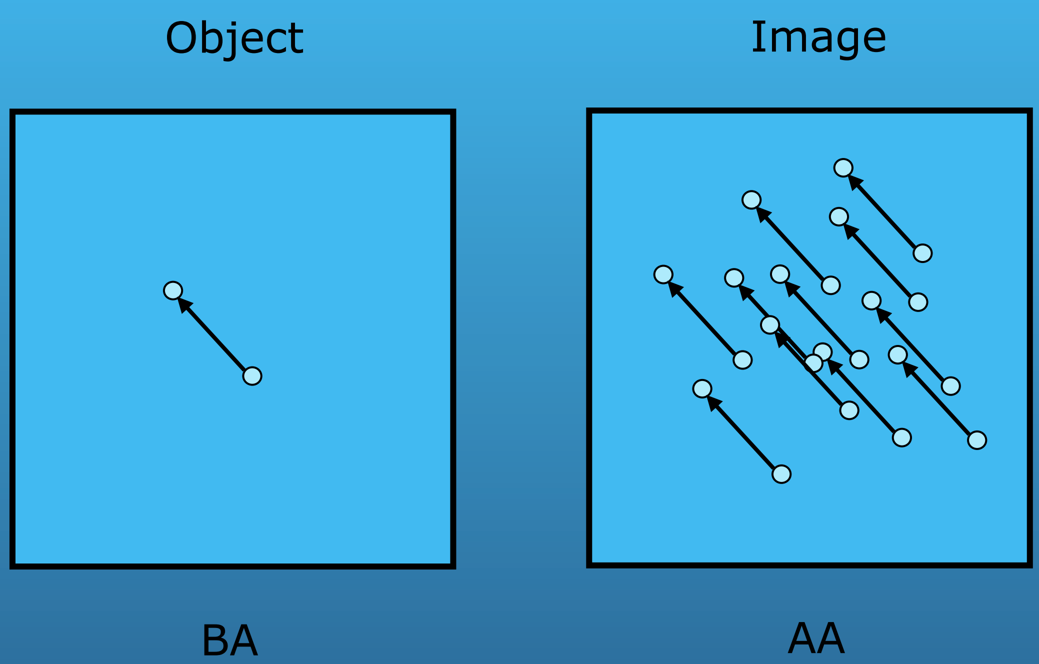
t=0.00s

t=0.05s

t=0.10s

t=0.15s

A binary star is a simple image morphology



SPECKLE IMAGE RECONSTRUCTION: Get back to BA picture from many AA images using image processing techniques.

Autocorrelation Analysis

$$\gamma(\mathbf{x}) = \iint I(\mathbf{x}')I(\mathbf{x}'+\mathbf{x})d\mathbf{x}' \xrightarrow{\text{FT}} FT\{\gamma(\mathbf{x})\} = \left|\hat{I}(\mathbf{u})\right|^2$$

$$I_i(x,y) = S_i(x,y) * O(x,y)$$

$$\hat{I}_i(u,v) = \hat{S}_i(u,v)\cdot\hat{O}(u,v)$$

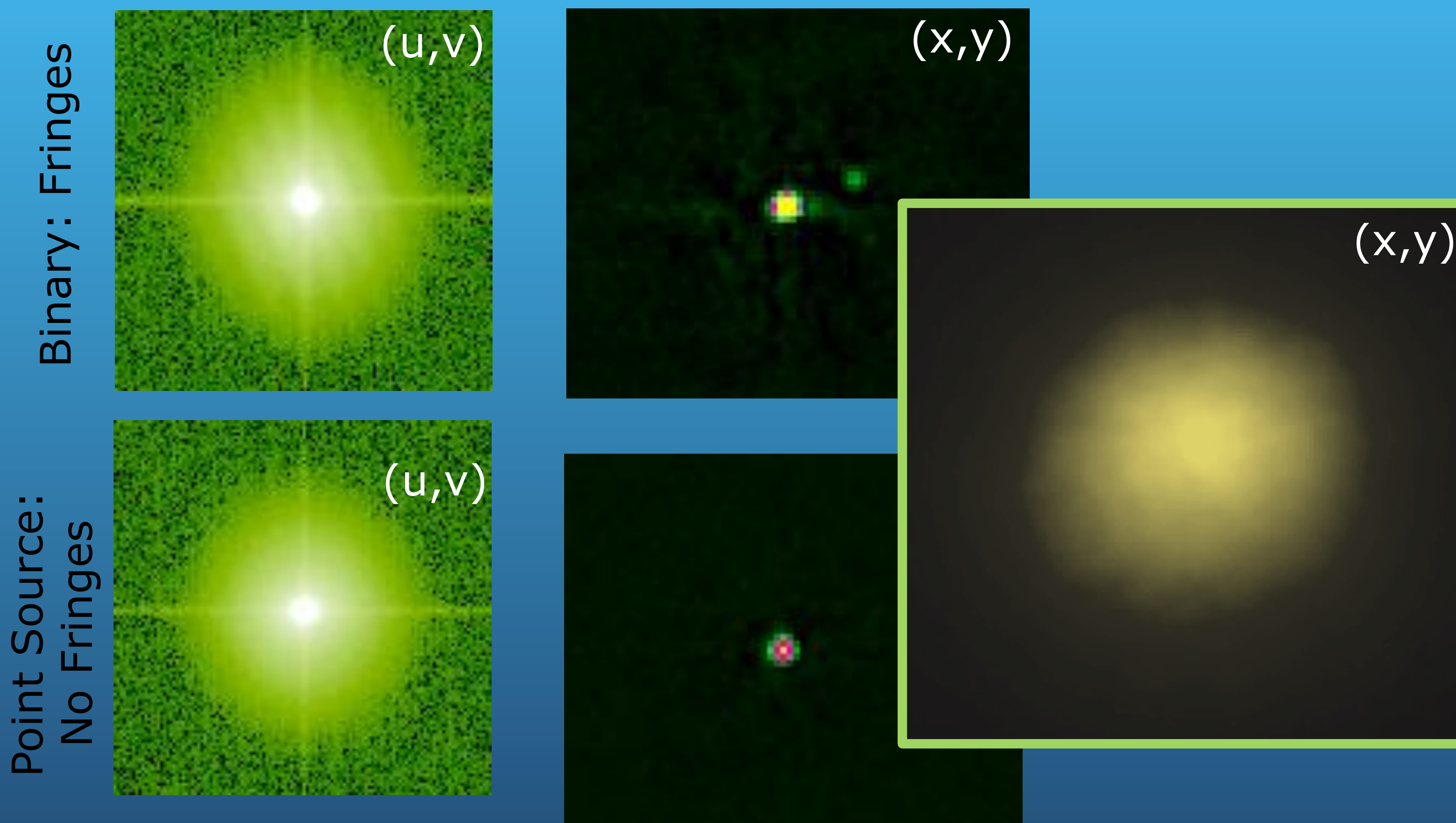
$$\left\langle \left|\hat{I}_i(u,v)\right|^2 \right\rangle = \left\langle \left|\hat{S}_i(u,v)\right|^2 \right\rangle \cdot \left|\hat{O}(u,v)\right|^2$$

Data
of the binary

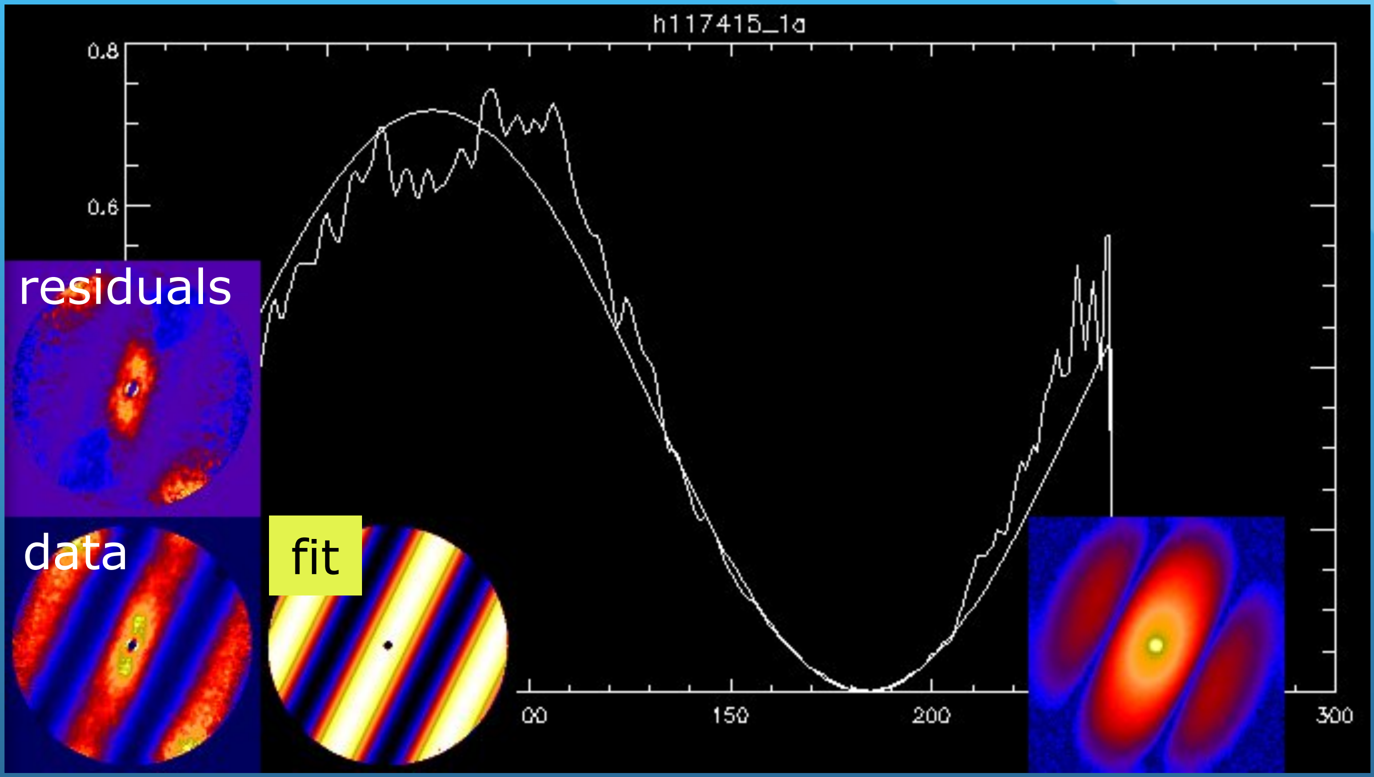
Data of a
Single star

$$\left|O(u,v)\right| = \sqrt{\frac{\left\langle \left|I(u,v)\right|^2 \right\rangle}{\left\langle \left|S(u,v)\right|^2 \right\rangle}}$$

For a Double Star, the Fringes are In the Fourier Plane.



Power spectrum of a binary



Power spectrum cannot give you an image:
- directed vector autocorrelation
- image reconstruction (bispectrum)

Spatial Frequency

- There will be power out to a certain radius in the power spectrum, corresponding to the diameter of the telescope in the (w,z) plane.
- Another way to say it is to use the Rayleigh criterion. Two point sources on the (x,y) plane can be as close together as $\theta_R=1.22\lambda/D$ and still be resolved. This is an angle, usually measured in arc seconds or milliarcseconds (mas). (There are 206265 arc seconds per radian).
  - Think of these point sources as successive peaks in a fringe pattern. Then the highest spatial frequency is given by $1/\theta_R$ cycles per arcsec. This is the limiting radius in the power spectrum.

Bispectral Analysis

Define triple correlation:

$$C(\mathbf{x}_1, \mathbf{x}_2) = \iint I(\mathbf{x}) I(\mathbf{x} + \mathbf{x}_1) I(\mathbf{x} + \mathbf{x}_2) d\mathbf{x}$$

FT is called the bispectrum, can be written:

$$\hat{C}(\mathbf{u}_1, \mathbf{u}_2) = \hat{I}(\mathbf{u}_1)\hat{I}(\mathbf{u}_2)\hat{I}^*(\mathbf{u}_1 + \mathbf{u}_2)$$

Sequence of speckle data frames contains diff. limited info:

$$\left\langle \hat{C}(\mathbf{u}_1, \mathbf{u}_2) \right\rangle = \hat{O}(\mathbf{u}_1)\hat{O}(\mathbf{u}_2)\hat{O}^*(\mathbf{u}_1 + \mathbf{u}_2)\left\langle \hat{S}(\mathbf{u}_1)\hat{S}(\mathbf{u}_2)\hat{S}^*(\mathbf{u}_1 + \mathbf{u}_2) \right\rangle$$

Let $u_1 = u$, $u_2 = \Delta u$, where Δu is small. Then, consider only the phase. Can show a point source should have zero phase. Then, $\arg\left\langle \hat{C}(\mathbf{u}, \Delta\mathbf{u}) \right\rangle = \varphi_O(\mathbf{u}) + \varphi_O(\Delta\mathbf{u}) - \varphi_O(\mathbf{u} + \Delta\mathbf{u})$

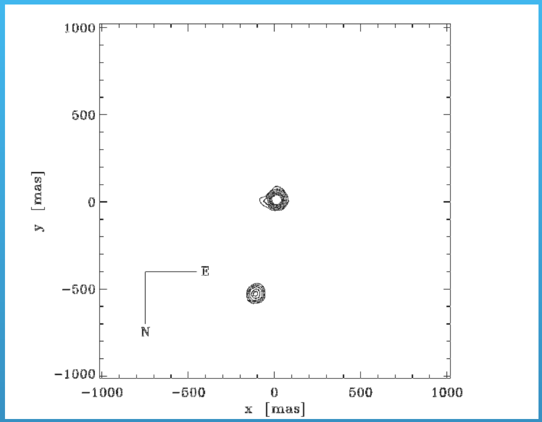
Bispectral Analysis, continued

Well, so $$\varphi_O(\mathbf{u}) - \varphi_O(\mathbf{u}+\Delta\mathbf{u}) = \arg\left\langle \hat{C}(\mathbf{u},\Delta\mathbf{u}) \right\rangle - \varphi_O(\Delta\mathbf{u})$$

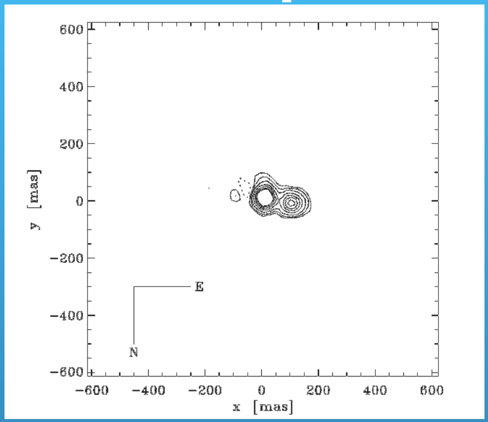
$$\frac{\varphi_O(\mathbf{u}+\Delta\mathbf{u}) - \varphi_O(\mathbf{u})}{\Delta\mathbf{u}} = -\frac{\arg\left\langle \hat{C}(\mathbf{u},\Delta\mathbf{u}) \right\rangle - \varphi_O(\Delta\mathbf{u})}{\Delta\mathbf{u}}$$

Thus, the bispectrum contains phase derivative information! By integrating, we obtain the phase, which can be combined with the modulus to obtain a diffraction-limited estimate of $\hat{O}(\mathbf{u})$.

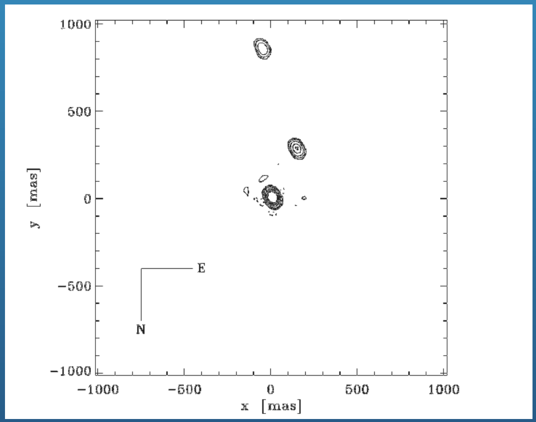
Reconstructed Images Examples.



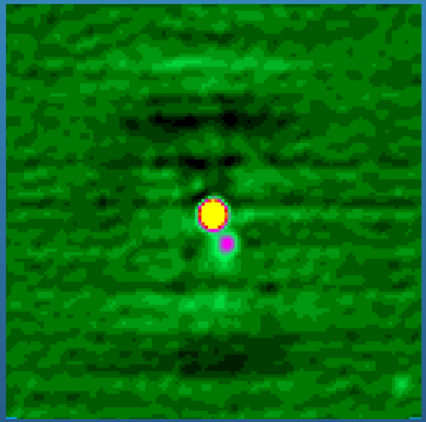
HIP 101769 = BU 151



HIP 098055 = YR 2



HIP 021730 = BU 1295AB + STF 566AB-C



HIP 085209 = HD 157948

Summary

- Speckle data frames contain high resolution information, albeit in a complicated way.
- The easiest way one can access this information by computing the autocorrelation of many frames of speckle data. This is a symmetric function on the image plane, does not contain phase information on the Fourier plane.
- To get the phase information missing in the autocorrelation, one can compute the triple correlation. This contains the derivative of the phase, which can be integrated to obtain the phase function.
- This allows one to “break the symmetry” of the autocorrelation, and produce an image reconstruction.