Potential Magnetic Field Extrapolation in Binary Star Systems

Volkmar Holzwarth[1], **Scott G. Gregory**[2]

[1]*Kiepenheuer-Institut für Sonnenphysik, Schöneckstr. 6, 79104 Freiburg, Germany*

[2]*School of Physics & Astronomy, University of St Andrews, St Andrews KY16 9SS, UK*

Abstract. For several decades the potential field source surface (PFSS) approximation has proven helpful in the description of the large-scale coronal magnetic fields of the Sun and of cool single stars. Here, we extend the PFSS technique to systems with two objects, to investigate the magnetic field structure of close binary stars. We describe the deviation of the two-centre extrapolation technique and demonstrate its applicability in the case of the close pre-main sequence binary system V4046 Sgr. Our results reveal a joint magnetosphere with complex field structures connecting both stellar components. The binary extrapolation method is also applicabe to the case of magnetic interaction between a host star and a close-in hot Jupiter.

1. Introduction

Magnetic fields have a decisive influence on the structural, dynamical and thermal properties of the coronae of cool stars. Since even in the case of the Sun direct observations of coronal magnetic fields are difficult to accomplish, extrapolation methods are frequently used to infer them from magnetic field distributions observed in the photosphere. Introduced by Schatten et al. (1969) for the case of the Sun, the potential field source surface (PFSS) extrapolation technique has since been used numerous times to describe the magnetospheres and wind zones of cool stars. It accounts for the existence of stellar winds through the assumption of magnetic fields becoming purely radial at some distance above the photosphere. Albeit more sophisticated extrapolation methods have been developed since (e.g., Wiegelmann & Sakurai 2012, and references therein), the PFSS approach remains significant, since the resulting magnetic field represents the stable, lowest-energy state of the magnetosphere which is consistent with boundary conditions given in the photosphere.

We extend the PFSS technique to close binary stars and demonstrate the applicability of the new method with the case of the close PMS binary V4046 Sgr, for which magnetic

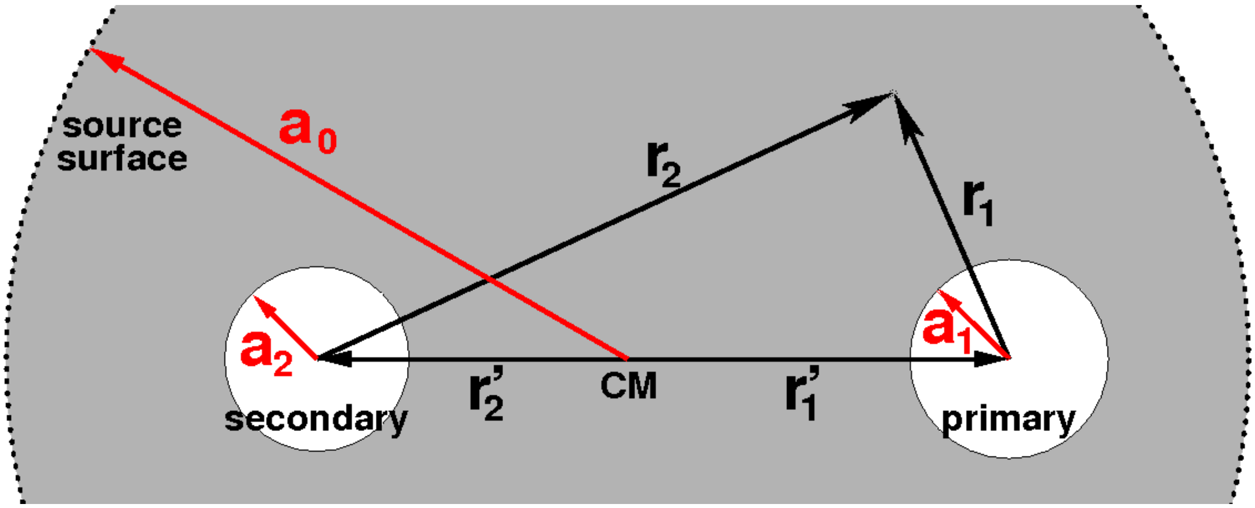

Figure .1: Schematic binary system. The region of interest (grey) is bounded at the outside by the source surface (dotted line) and inside by the stellar surfaces.

surface maps for both stellar components have been reconstructed based on Zeeman-Doppler imaging observations (Donati et al. 2011). Our extrapolation method is applicable also to star-planet and planet-moon systems, and easily expandable to N-body systems.

2. Binary field extrapolation technique

We consider a binary system consisting of a primary star, S_1, with radius a_1 located (with respect to the center of mass, CM) at point $\vec{r}_1'$ and a secondary star, S_2 with radius a_2 at point $\vec{r}_2'$. The source surface S_0, with radius a_0 and centred in CM, marks the location at which the magnetic field becomes purely radial through the effect of winds from both stellar components. In the case of the Sun, a standard value is $a_0 = 2.5\,R_\odot$ (e.g. Schatten et al. 1969), whereas for single stars the position of the source surface is empirically determined through, for example, through a comparison of reconstructed emission measures with empirical values based on X-ray observations (Jardine et al. 2002).

The region of interest, shaded grey in Fig. .1, is taken to be current free. The magnetic field, $\vec{B} = -\nabla\Psi$, is therefore given by the gradient of a scalar potential function, Ψ, which, owing to the solenoidal condition $\nabla \cdot \vec{B} = 0$, satisfies the Laplace equation, $\nabla^2\Psi = 0$. The joint magnetesphere of the binary system is thus determined by solving the Laplace equation, subject to boundary conditions specified on the surfaces of both stellar components and on the source surface.

The potential Ψ is expanded in terms of regular and irregular solid spherical harmonics (SSH),

$$R_l^m(\vec{r}) = (-1)^m \frac{r^l}{(l+m)!} P_l^m(\cos\theta) e^{im\phi} \tag{1}$$

$$S_l^m(\vec{r}) = (-1)^m \frac{(l-m)!}{r^{l+1}} P_l^m(\cos\theta) e^{im\phi} \ , \tag{2}$$

respectively (Steinborn & Ruedenberg 1973), around the origins $\vec{r}_k'$ of the spheres S_k, where we take into account that $\vec{r} = \vec{r}_k' + \vec{r}_k$ (see Fig. .1):

$$\Psi(\vec{r}) = \sum_{l,m} c_{lm}^{(0)} R_l^m(\vec{r}_0) + \sum_{l,m,k>0} c_{lm}^{(k)} S_l^m(\vec{r}_k) \tag{3}$$

The second term on the right hand side of Eq. (3) comprises the magnetic moments of the two stellar components, whereas the the first term contains the magnetic moments arising from the boundary condition at the source surface; in mathematical terms, the later is realised through the assumption of mirror sources located beyond the source surface, which in concert with the magnetic moments of the two stars result in a radial magnetic field at S_0.

The boundary conditions at S_1, S_2, and S_0 specify expansion coefficients, $c_{lm}^{(k)}$. For their determination, we make use of SSH translation theorems (e.g. Steinborn & Ruedenberg 1973),

$$R_l^m(\vec{r}_1 + \vec{r}_2) = \sum_{\lambda,\mu} (R|R)_{\lambda l}^{\mu m}(\vec{r}_1) R_\lambda^\mu(\vec{r}_2) = \sum_{\lambda,\mu} R_{l-\lambda}^{m-\mu}(\vec{r}_1) R_\lambda^\mu(\vec{r}_2) \tag{4}$$

$$S_l^m(\vec{r}_> + \vec{r}_<) = \sum_{\lambda,\mu} (S|R)_{\lambda l}^{\mu m}(\vec{r}_>) R_\lambda^\mu(\vec{r}_<) = \sum_{\lambda,\mu} (-1)^{\lambda-\mu} S_{l+\lambda}^{m-\mu}(\vec{r}_>) R_\lambda^\mu(\vec{r}_<) \tag{5}$$

$$S_l^m(\vec{r}_> + \vec{r}_<) = \sum_{\lambda,\mu} (S|S)_{\lambda l}^{\mu m}(\vec{r}_<) S_\lambda^\mu(\vec{r}_>) = \sum_{\lambda,\mu} (-1)^{\lambda+\mu-l-m} R_{\lambda-l}^{m-\mu}(\vec{r}_<) S_\lambda^\mu(\vec{r}_>)\ , \tag{6}$$

which describe the contribution of a magnetic moment (l, m) of an expansion to the magnetic moment (λ, μ) of a translated expansion shifted by translation vector $\vec{r}_1$. Note that in the case of irregular SSH, one has to distinguish between regions which are closer, Eq. (5), and further, Eq. (6), away from the shifted centre of expansion than the applied translation vector, that is $|\vec{r}_<| < |\vec{r}_>|$.

The contributions of the stellar magnetic moments to the expansion of the potential function on the source surface are described through Eq. (6), that is

$$\Psi(\vec{r}_0)\big|_{S_0} = \sum_{l,m} c_{lm}^{(0)} R_l^m(\vec{r}_0) + \sum_{l,m,k>0} c_{lm}^{(k)} S_l^m(\vec{r}_0' - \vec{r}_k' + \vec{r}_0) \tag{7}$$

$$= \sum_{l,m} c_{lm}^{(0)} R_l^m(\vec{r}_0) + \sum_{l,m,k>0} c_{lm}^{(k)} \sum_{\lambda,\mu} (S|S)_{\lambda l}^{\mu m}(\vec{r}_{k0}') S_\lambda^\mu(\vec{r}_0) \tag{8}$$

with $\vec{r}_{k0}' = \vec{r}_0' - \vec{r}_k'$. The boundary condition of magnetic fields being purely radial implies $\Psi(\vec{r}_0)\big|_{S_0} = \text{const}$. For the sake of simplicity, we set $\Psi(\vec{r}_0)\big|_{S_0} = 0$, which yields the determining equations for the coefficients

$$c_{lm}^{(0)} = -\frac{1}{a_0^{2l+1}} \sum_{l',m',k>0} (S|S)_{ll'}^{mm'}(\vec{r}_{k0}') c_{l'm'}^{(k)} \tag{9}$$

in terms of the stellar magnetic moments $c_{l'm'}^{(k)}, k > 0$.

Inserting Eq. (9) and Eqs. (4-5) in Eq. (3) yields the expansion of the potenial on the surface S_j of stellar component j:

$$\Psi(\vec{r}_j)\big|_{S_j} = \sum_{l,m} c^{(j)}_{lm} S^m_l(\vec{r}_j) + \sum_{\lambda,\mu,j\neq k>0} c^{(jk)}_{\lambda\mu} R^{\mu}_{\lambda}(\vec{r}_j) \tag{10}$$

Whereas the first term on the right side of Eq. (10) accounts for the contributions of the magnetic moments of star j, the second term, with the new coefficients

$$c^{(jk)}_{\lambda\mu} = -\sum_{l',m'} c^{(j)}_{l'm'} W^{\mu m'}_{\lambda l'}(\vec{r}_{0j}, \vec{r}_{j0}, a_0) \tag{11}$$

$$+ \sum_{l,m,k\neq j} c^{(k)}_{lm} \left[(S|R)^{\mu m}_{\lambda l}(\vec{r}_{kj}) - W^{\mu m}_{\lambda l}(\vec{r}_{0j}, \vec{r}_{k0}, a_0) \right] \tag{12}$$

subsumes the contributions of both magnetic moments of the companion star and influence of binary winds (i.e. the source surface). In Eq. (12), the abbreviation

$$W^{m_R m_S}_{l_R l_S}(\vec{r}_R, \vec{r}_S, a) = \sum_{l,m} (R|R)^{m_R m}_{l_R l}(\vec{r}_R)\,(S|S)^{m m_S}_{l l_S}(\vec{r}_S) \frac{1}{a^{2l+1}} \tag{13}$$

has been introduced. The boundary conditions on the stellar surfaces are given by maps of the radial magnetic field component. The coefficients, $B^{(j)}_{l'm'}$, resulting from the expansion of these maps in terms of spherical harmonics, determine the magnetic moments $c^{(j)}_{lm}$:

$$\begin{aligned} B^{(j)}_{l'm'} &= \frac{l'+1}{a_j^{l'+2}} c^{(j)}_{l'm'} + l' a_j^{l'-1} \sum_{l,m} W^{m'm}_{l'l}(\vec{r}_{0j}, \vec{r}_{j0}, a_0) c^{(j)}_{lm} \\ &\quad - l' a_j^{l'-1} \sum_{k\neq j,l,m} \left[(S|R)^{m'm}_{l'l}(\vec{r}_{kj}) - W^{m'm}_{l'l}(\vec{r}_{0j}, \vec{r}_{k0}, a_0) \right] c^{(k)}_{lm} \end{aligned} \tag{14}$$

In the case of a binary system, a solution of the linear system of equations

$$\begin{pmatrix} B^{(1)}_{l',m'}/a_1^{l'-1} \\ B^{(2)}_{l',m'}/a_2^{l'-1} \end{pmatrix} = \begin{pmatrix} \mathcal{M}_{11} & \mathcal{M}_{12} \\ \mathcal{M}_{21} & \mathcal{M}_{22} \end{pmatrix} \begin{pmatrix} c^{(1)}_{l,m} \\ c^{(2)}_{l,m} \end{pmatrix} \tag{15}$$

with

$$\mathcal{M}_{jk} = \begin{cases} \delta_{ll'}\delta_{mm'}(l'+1)a_j^{-(2l'+1)} + l' W^{m',m}_{l',l}(\vec{r}_{0j}, \vec{r}_{j0}, a_0) & , \quad j = k \\ -l'\,(S|R)^{m',m}_{l',l}(\vec{r}_{kj}) + l' W^{m',m}_{l',l}(\vec{r}_{0j}, \vec{r}_{k0}, a_0) & , \quad j \neq k \end{cases} \tag{16}$$

determines the expansion coefficients and, thus, the structure of the joint magnetosphere.

3. Example: V4046 Sgr

We demonstrate the applicabality of our method through the reconstruction of the joint magnetosphere of the young ($\sim$ 13 Myr) pre-main sequence binary V4046 Sgr. Figure .2

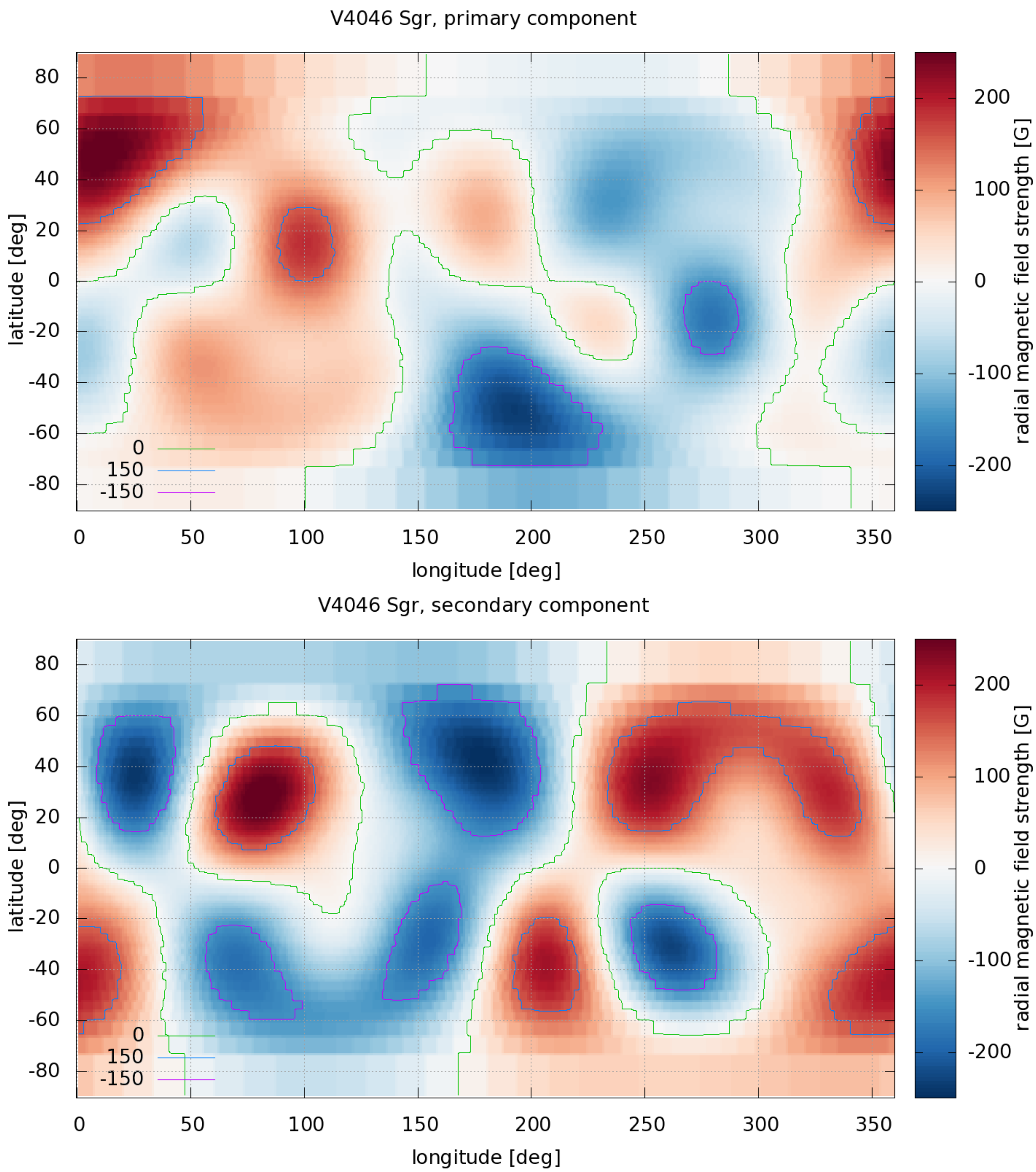

Figure .2: Surface distributions of the radial magnetic field strength on the primary (top) and secondary (bottom) component of the pre-MS binary V4046 Sgr, reconstructed from Zeeman-Doppler imaging observations (Donati et al. 2011).

shows surface maps of the radial magnetic field strength of both stellar components, based on spectropolarimetric observations using ESPaDOnS at the CFHT in 2009 (Donati et al. 2011; Gregory et al. 2014).

The system parameters (cf. Fig. .1) used for the reconstruction of the magnetosphere are given in Table .1. The source surface radius of a binary system is, as yet, observationally

surface S_k	k	position $x'_k[\,R_\odot]$	radius $a_k[\,R_\odot]$
source	0	0	10.0
primary	1	4.3	1.12
secondary	2	-4.5	1.04

Table .1: Parameter combination for the potential magnetic field extrapolation of V4046 Sgr, shown in Fig. .3; positions, $\vec{r}'_k = (x'_k, 0, 0)^T$, along the x-axis are relative to centre of mass, CM, of binary system (Fig. .1).

unconstrained, so that $a_0 = 10\,R_\odot$ is somewhat arbitrarily. This value has been chosen based on the assumption that, for the hemispheres pointing away from the companion, the source surface is in a similar distance as for respective single star models.

Our reconstruction reveals complex magnetic field structures in the binary system (Fig. .3). The magnetic field in the vicinity of each star is dominated by tilted dipoles, somewhat modified by the influence of higher magnetic moments. Of particular interest are magnetic field structures connecting both stellar components; some 'S'-shaped field lines can be identified which inter-connect the dayside hemisphere of one component with the night side of the other.

4. Discussion

Close, late-type binary stars, such as RS CVn and BY Dra systems, are among the most active stellar objects we know. Their combination of rapid rotation and convective motions entails pronounced magnetic activity signatures which are discernible from the visible to the X-ray spectral range. The interpretation of, e.g., the amount and variability of coronal X-ray emission or the location of high-energetic flaring, requires, however, some notion of the magnetic field underlying their atmospheres. To this end, we extend the PFSS technique to close binaries to provide, in conjunction with Zeeman-Doppler imaging observations of photospheric magnetic fields, approximations of their joint magnetospheres.

More advanced (e.g. non-linear force-free field) extrapolation techniques are frequently applied locally, for instance to active regions on the Sun, where high-resolution observations permit to set up sophisticated boundary conditions. In the stellar case, however, the less-demanding PFSS approach seems adequate, because detailed observations of small-scale magnetic field distributions are unavailable. Disregarding electric currents and dissipation mechanisms, the PFSS approximation provides the lowest-energy, and therefore stable, state of the global field configuration. This allows us to address question related to the distribution of open, closed, and inter-connecting magnetic flux on the surfaces of binary stars, the inter-

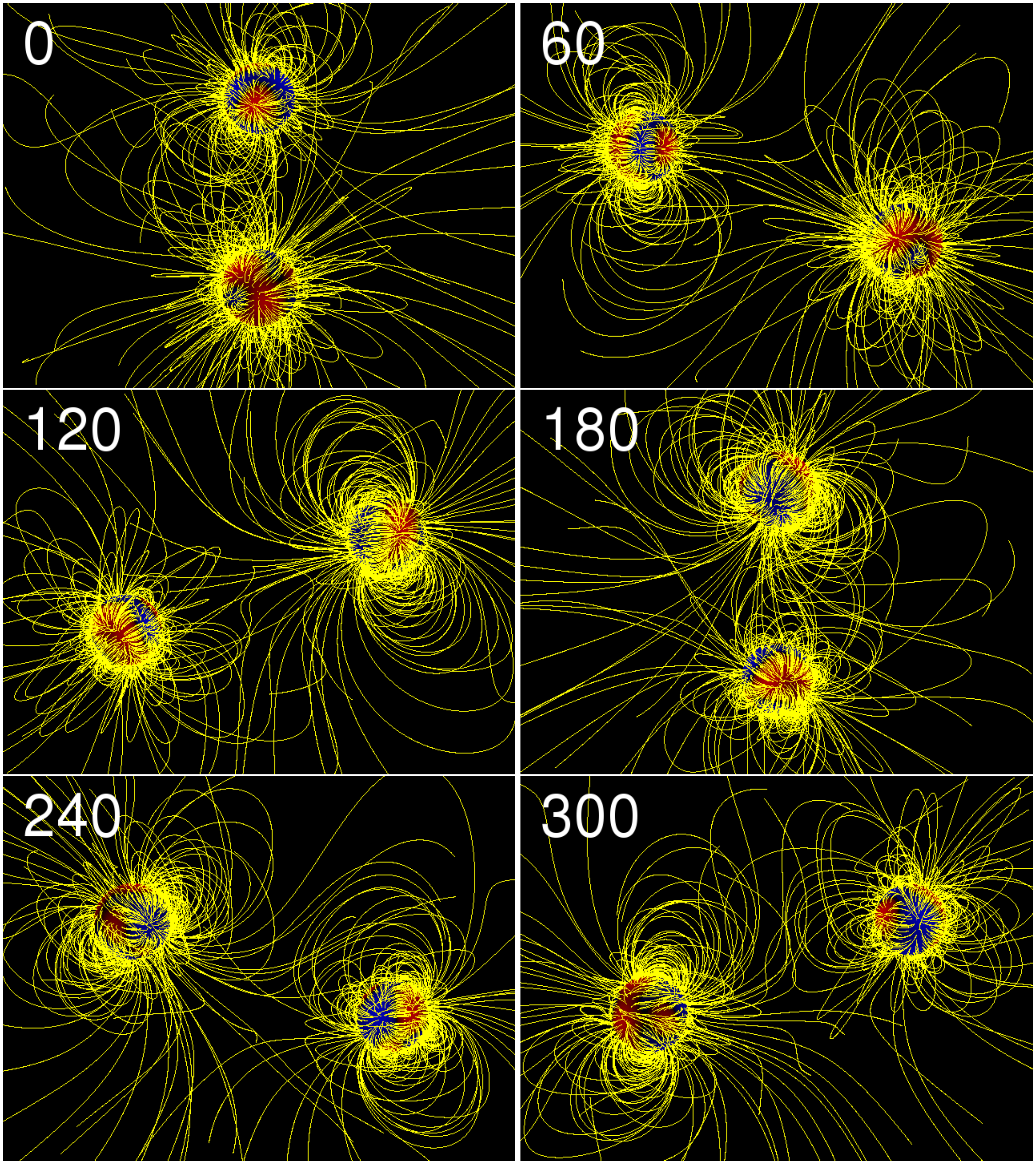

Figure .3: Joint magnetosphere of the pre-MS binary V4046 Sgr. Field line tracings seen from consecutive azimuthal viewing angles, ϕ; for $\phi = 0$ (top-left), the primary is in the lower half of the picture. Colour shading of the stellar surfaces indicate strength and polarity of the radial magnetic field component.

binary field structure and locations of activity, and on the mass transfer along connecting field lines. Our preliminary results for V4046 Sgr reveal, for instance, complex field structures which connect different hemispheres of both stellar components.

Our method of potential magnetic field extrapolation is applicable to detached binary star systems, single stars with exoplanets, and single stars with asymmetric winds (i.e. an off-centred source surface). Objects which deviate significantly from spherical geometry such as semi-detached or contact systems are currently excluded, since any rotational or tidal deformations are neglected. Yet an extension of our method to systems with $k_{\max} > 2$ objects is straightforeward, so that it can easily be applied to N-body problems involving potential fields of different kind, such as source-free magnetic, electric, or gravitational fields.

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