Reproduction of the Wilson-Bappu Effect Using PHOENIX

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Abstract.

We use the versatile PHOENIX atmospheric modeling code in its version, which includes a gravity-scaled chromosphere above the temperature minimum to model the Ca II K emission line profile for solar-type stars, all with $T_{\text{eff}} = 5780$ K and same turbulence broadening, only with different surface gravities. Models, which produce the modest emission observed in relatively inactive stars, reproduce the Wilson-Bappu effect (WBE) in absolute terms, i.e. the emission line-widths grow with lower gravity consistent with $\Delta W \propto g^{-0.17}$ in the range of $\log g = 5.0$ to 3.5. Further modeling is in process to include lower gravities.

In the solar case, which we used as a first test, we find the temperature minimum (over height, single component) for a relatively inactive Sun to reach down to 3930 K. The respective PHOENIX model ($\log g = 4.4$) matches width and typical flux of the chromospheric Ca II emission of a nearly inactive Sun, as observed with the Hamburg robotic telescope (see Fig. 1). For comparison, the quiet Sun model C of Vernazza et al. (1981) had a temperature minimum of 4170 K.

1. The WBE - 57 years on

In their classic paper, Wilson & Vainu Bappu (1957) announced that higher luminosity cool stars show a systematically larger width $W$ of their chromospheric emission in the core of the Ca II H &K lines than those of a lower luminosity, and that there was a linear relationship between absolute magnitude and log $W$. In the decades to follow, and to the present day, this Wilson-Bappu effect receives a lot of interest, for two reasons: (1) as a distance indicator, and (2) for reflecting some fundamental chromospheric physics.
Nevertheless, it took 16 years until Reimers (1973) showed empirically, that the apparent dependence of \( W \) on the visual absolute magnitude must stem from a physical relation between \( W \) and the surface gravity, approximately \( W \propto g^{-0.2} \). On the same grounds, Reimers (1973) argued that there also was a physical dependence on \( T_{\text{eff}} \), which would however be masked by the increase of BC towards cooler stars. Modern empirical studies (a recent example is Park et al. (2013)) suggest a similar, slightly shallower gravity dependence of about \( g^{-0.17} \).

Finally, Ayres et al. (1975) explained the Wilson-Bappu effect as a natural consequence of a chromosphere in hydrostatic equilibrium (not considering any effects of magnetic fields), where consequently the column density scales with \( N \propto g^{-1/2} \), and by the strong saturation of the Ca II H&K emission line profile. Assuming the simple relation of line growth in damping wings (\( \tau_{\text{line}} \propto N^{1/2} \)), Ayres and Linsky got a relation of \( W \propto g^{-0.25} \), close enough for the simplification made. More realistically, it is the transition between the slower growing Doppler core and the damping wing, on which line growth depends. In this case the line profile function is best given by a Voigt profile. Its slower growth explains why the observed relation has a shallower \( g \)-dependence (see below).

### 2. \( g \)-scaled chromospheric models

The PHOENIX code normally is only a photospheric model, i.e. without temperature reversal. For this work, we used our PHOENIX version, which includes a chromospheric 1D model in hydrostatic equilibrium, with the same essential physics as summarized above, and which therefore simply scales with surface gravity (\( N \propto g^{-1/2} \), \( n \propto g^{1/2} \)). See Fig. 2 for our model of the solar case. It also matches the solar \( W \) of 0.44 Å.

Assuming a simple 1-temperature-component atmosphere, we use the depth of the solar Ca II K line around the emission core to precisely determine the best-matching stratification around the temperature minimum (Fig. 2), whose temperature stratification is shown in Fig. 3. For the total emission, we were guided by our own observations of the inactive Sun in 2009 and the moderately active Sun earlier this year (see Fig. 1), where the broadening effect of the instrumental profile of 0.2 Å has to be taken into account.

### 3. Reproduction of the WBE from first principles

Using the solar effective temperature, we then computed models with different gravity in order to see, if these would reproduce the Wilson-Bappu effect. A practical problem occurs in that the shallow basal flux emission is too smeared out at already \( \log g = 3.5 \). Consequently, we needed to make the bottom of the chromosphere (just above the temperature minimum) a little warmer to mimic the emission of modestly active stars – which in fact represent the stars observed for the WBE. But the equilibrium conditions allow only for a
Figure 1: Spectra of TIGRE (formerly Hamburg Robotic Telescope) and its HEROS spectrograph (R=20,000 resolution) show the solar Ca II K line core at its basal flux level in 2009 (S=0.15, Schröder et al. (2012)) and at an only moderately active level in 2014 (S=0.17). Note that the instrumental profile causes a broadening of about 0.2 Å, which levels the computed emission core of Fig. 2 significantly.

small margin on this. See Fig. 4 for some actual line profiles and Tab. 1 for the set of line width measurements (flank to flank, at half peak).

We do not adjust any other parameter than surface gravity to obtain our emission line profiles (i.e., keeping turbulence velocities alike) – hence, these are produced from first principles and so represent a good test of the WBE explanation given by Ayres & Linsky nearly 40 years ago. But now we can take precise care of the line saturation by using a Voigt profile. As a result, our line widths reproduce the observed WBE gravity-dependence with an exponent of -0.17 (rather than -0.25) very well and in absolute terms.
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Figure 2: Low activity chromospheric emission of the solar PHOENIX model \((T_{eff} = 5780\, \text{K}, \log(g) = 4.4)\) with the temperature stratification of Fig. 3. The half-peak width of 0.44 Å (or 34 km/s) matches the solar, instrumental-profile corrected width of the solar K2 emission.

<table>
<thead>
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<th>(\log g)</th>
<th>5.0</th>
<th>4.4</th>
<th>4.0</th>
<th>3.5</th>
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<td>(W/\AA)</td>
<td>0.37</td>
<td>0.44</td>
<td>0.50</td>
<td>0.55</td>
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</tbody>
</table>

Table 1: Line width \((W)\) (flank to flank, half peak) of the chromospheric Ca II K emission computed by PHOENIX models for different surface gravity \(g\) and \(T_{eff} = 5780\, \text{K}\). \(W\) matches the solar case and its growth with lower \(g\) is fully consistent with the observed relation \(W \propto g^{-0.17}\).

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References

Figure 3: Radial temperature (1-component) stratification in height of the solar PHOENIX model, which produces the chromospheric Ca II K line emission of Fig. 2 and a generally good match of the photospheric line spectrum of the Sun. The temperature minimum reaches to 3930 K.


Figure 4: Three PHOENIX models with $T_{\text{eff}} = 5780$ K for surface gravities of $\log(g) = 5.0$ (left), $\log(g) = 4.0$ (right), and 3.5 (bottom). The slow increase of the width $W$ with lower gravity (see Table 1) reproduces well the observed WBE, $W \propto g^{-0.17}$.
A few of the 296 posters that were up all week at Cool Stars 18.