Stellar Masses

Arguably the mass of a star is its most important property. In this article we will answer four questions concerning stellar masses. How do the properties of stars depend upon their masses? What is the smallest and largest mass possible for a star, and why? How do we determine the masses of stars? What distribution of stellar masses occurs when stars form, and why?

Dependence of Other Stellar Parameters on Mass

The *Russell-Vogt theorem* states that if we know a star's mass and its chemical composition, we can use the laws of physics to determine all of its other properties: its luminosity, its radius, its temperature and density profiles, and how these properties change with time. (We know that this is a slight simplification; for instance, the amount of net angular momentum will also affect a star's structure and evolution). Compared to the possible range of masses a star may have (0.08–150 $M_\odot$), there is only modest variation possible in the initial composition, and thus it is primarily a star's mass at birth which determines the basic essentials of its structure and future life.

Some of the properties of stars are given in Table 1 as a function of stellar mass for stars on the *main-sequence*, the core H-burning phase that accounts for 90% of a star's life. These values have been taken from stellar models computed with a composition that is initially solar. We list the stellar parameters at the beginning and end of the main-sequence lifetimes, except for the lowest mass stars, for which we adopt the parameters corresponding to an age of 1 Gyr, by which time these stars are stably burning hydrogen.

Generally the behavior of the stellar parameters with stellar mass is quite different for the higher mass stars (25–120 $M_\odot$) than for solar-type stars (0.8-1.2 $M_\odot$). The dependence of luminosity on stellar mass is shown in Figure 1. This *mass-luminosity relationship* is considered one of the most fundamental descriptions of stellar properties; the ability to reproduce this by stellar models was one of the great vindications of theory. Eddington first demonstrated that radiative diffusion in stars requires that the stellar luminosity will depend upon mass roughly as the fourth power; i.e., $L \sim M^4$. However, it is clear from Figure 1 that no single exponent describes the dependence of luminosity on mass over the entire range of stellar masses. If we consider different mass-ranges we would find that the following are good approximations:

$$L \sim M^{1.6} (M \approx 100 M_\odot)$$

$$L \sim M^{3.1} (M \approx 10 M_\odot)$$

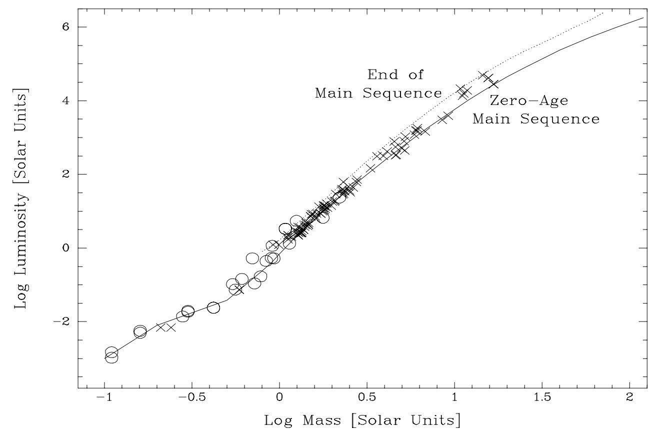

Figure 1: The mass-luminosity relationship as predicted by stellar models is shown by the solid line for stars with zero age, and by the dotted line for stars at the end of their main-sequence lifetimes, for masses of 0.8 $M_\odot$ ($\log M = -0.1$) and greater. Below that mass, the curve shown is for models with an age of 1 Gyr, as stars of this age have begun to burn hydrogen stably with constant luminosity. (The vast majority of low-mass stars will be at least that old.) The points show the masses and luminosities of "real" stars for comparison, with the crosses denoting the best determinations from double-lined spectroscopic binaries, and the open circles denoting the best determinations from visual binaries.

$$L \sim M^{4.7} (M \approx 1 M_\odot)$$

and

$$L \sim M^{2.7} (M \approx 0.1 M_\odot)$$

The reason for the drastic changes seen in the mass-luminosity relation with mass are primarily due to the different opacity sources at work. At the high interior temperatures that characterize high mass stars, all of the atoms are fully ionized and scattering of X-rays from free electrons dominates the opacity, with no temperature dependence. At lower temperatures, atoms are only partially ionized, and there is a strong temperature dependence in the number of ions. At the very cool temperatures that characterize the lowest mass stars, molecular hydrogen (H_2) forms, removing the dominant opacity source for solar-type stars, H^-.

Stellar lifetimes τ_{ms} as a function of mass also show a marked change from solar-type star to high-mass stars, as evidenced by Table 1. For solar-type stars the main-sequence lifetime changes rapidly with mass, while for higher mass stars the change is far more modest with mass. For most stars, roughly the same fraction of a star's mass (10%) is involved in nuclear burning regardless of mass, and so the relative main-sequence lifetime $\tau_{\rm ms}$ will be roughly proportional to the mass (the amount of fuel) and inversely propor-

Table 1: Properties of Main-Sequence Stars as a Function of Stellar Mass

Mass	τ_{ms}	Beginning of Main-Sequence				End of Main-Sequence			
		T_{eff} (°K)	Spectral Type	$\log L/L_\odot$	Rad. ($R_\odot$)	T_{eff} (°K)	Spectral Type	$\log L/L_\odot$	Rad. ($R_\odot$)
120 $M_\odot$	2.56 Myr	53,300	O3 V	+6.25	16	32,900	O9 I	+6.34	48
60 $M_\odot$	3.45 Myr	48,200	O4 V	+5.73	10	12,000	B7 I	+5.99	230
25 $M_\odot$	6.51 Myr	37,900	O8 V	+5.29	6.5	29,000	B0 I	+5.29	18
12 $M_\odot$	16.0 Myr	28,000	B0.2 V	+4.01	4.3	24,400	B0.5 I	+4.46	9.5
5 $M_\odot$	94.5 Myr	17,200	B5 V	+2.74	2.7	15,100	B5 I	+3.15	5.5
2.5 $M_\odot$	585 Myr	10,700	B9 V	+1.60	1.8	9,000	A2 III	+1.92	3.8
1.25 $M_\odot$	4.91 Gyr	6,380	F5 V	+0.32	1.2	6,070	G0 V	+0.66	1.9
1.0 $M_\odot$	9.84 Gyr	5,640	G8 V	−0.16	0.9	5,790	G2 V	+0.22	1.3
0.8 $M_\odot$	25.0 Gyr	4,860	K2 V	−0.61	0.7	5,360	K0 V	−0.09	1.1
0.5 $M_\odot$	100 Gyr	3,890	M0 V	−1.42	0.4	—	—	—	—
0.2 $M_\odot$	4,000 Gyr	3,300	M4 V	−2.2 :	0.2	—	—	—	—
0.1 $M_\odot$	10,000 Gyr	2,900	M7 V	−3.0 :	0.1	—	—	—	—

tional to the luminosity (how quickly the fuel is consumed); i.e., $\tau_{ms} \sim M/L$. Given the mass-luminosity relations above, we can thus estimate the dependence of lifetime on mass as $\tau_{ms} \sim M^{-3.7}$ for solar-type stars, and $\tau_{ms} \sim M^{-0.6}$ for very high-mass stars. This rule-of-thumb breaks down for the lowest mass stars, as the stars are fully convective, and the hydrogen-burning main-sequence lasts a good deal longer than one would expect. As shown in Table 1, a 0.1 $M_\odot$ star will last 10 *trillion* years (1.0×10^{13} yr) in a core-H burning phase, roughly 1000 times as long as the sun will, rather than the factor of 100 that one would expect, since the entire star provides the nuclear fuel.

During the main-sequence the highest mass stars lose a significant fraction of their mass due to stellar winds. A star that begins life with 120 $M_\odot$ star will lose 50 $M_\odot$ (40%) of its mass, while a 60 $M_\odot$ will lose 12 $M_\odot$ (20%), by the end of its main-sequence life. Below 25 $M_\odot$ the amount of mass lost during main-sequence evolution is negligible, although stellar winds do effect the evolution of even solar-type stars by carrying off angular momentum. Such mass-loss is expected to scale with metallicity and thus will be less significant in galaxies of lower metallicity.

It is inappropriate to speak of a spectral-type to mass relationship for higher-mass stars: stellar evolution results in a progression from higher effective temperatures to cooler during the core-H burning lifetime, and during this evolution stars of different masses will pass through a particular spectral type "stage". For lower mass main-sequence stars this is not true, and there is only a slight change of spectral-type with evolution (i.e., little change of the effective temperature). For example, a star which is spectroscopically classified as "O4 V" star may be a zero-age 60 $M_\odot$ star, or a slightly older (0.5 Myr) 85 $M_\odot$ star, but all stars of spectral type G2 V will have a mass roughly that of the sun's.

Range of Stellar Masses: the Lowest and Highest Mass Stars

The masses of stars span the range of 0.08 to 150 (or more) times the mass of the sun.

At the low mass end, the 0.08 $M_\odot$ limit is set by the stellar core not being hot enough to ignite hydrogen stably. Objects with masses slightly below this limit are called brown dwarfs, and are "star-like" in the sense that nuclear burning of deuterium occurs in their core. Below a mass of 0.015 $M_\odot$ (roughly 16 times the mass of Jupiter) not even deuterium burning can occur, and these objects are perhaps best called planets. Thus there is a natural lower limit to what constitutes a star, although we expect that the mass function (discussed below) should be unaware of this division. We note that the distinction between brown dwarfs and bona fide stars is subtle in the following sense: both low-mass stars and brown dwarfs burn primordial deuterium at first, but a "real" star will "eventually" settle down to stable H-burning; we expect that in the case of a 0.08 $M_\odot$ star that this will take approximately a billion years (1 Gyr).

At the other extreme, we do not understand what, if anything, limits how large a mass that a star may have. At one time it was thought that radiation pressure acting on grains limited how large a star could form, but we now understand that disks play an important role in the formation of stars. There may be sufficient shielding by the inner part of the disk to mitigate the effects of radiation pressure. It is not clear at this time what role the mergers of protostellar clumps may play in the formation of stars. If the role is appreciable, then there may be *no* natural limit to how massive a star may actually form.

Even if star-formation processes fail to limit the mass of a star, other processes may. Eddington proposed in 1926 that stars more massive than some amount would be pulsationally unstable, and should blow off their outer layers, thus limiting their mass. Early estimates of this limit were as low as 60 $M_{\odot}$. Modern estimates, however, place this limit as high as 440 $M_{\odot}$, although this is still based upon the same classical perturbation linerarization methods used by Eddington. Recent "nonlinear" analysis (i.e., direct numerical integration of the equations of stellar structure) suggest that the mass-loss from such instabilities would only be comparable to the mass-loss of radiatively-driven stellar winds in any event.

In this context it is interesting to note that the highest mass stars we know do all show signs of prodigious mass loss. The highest mass main-sequence stars known are located in the R136 cluster at the heart of the 30 Doradus Nebula in the Large Magellanic Cloud. These stars have masses which have been conservatively estimated as being as high as 155 $M_{\odot}$. Spectroscopically the eight most massive of these stars show evidence of extremely high mass-loss rates (mimicking the appearance of Wolf-Rayet stars), and so one could argue that indeed these stars are not "stable" in the sense that they are losing a considerable amount of matter.

The luminosities of the most luminous R136 stars are $10^{6.6} L_{\odot}$. Other stars which are of comparable luminosity include HD 5980, a Wolf-Rayet star in the SMC; η Carina, a Luminous Blue Variable (LBV) in the Milky Way; Sk$-67°$ 211, an O3 III star in the LMC; and the Pistol Star, an LBV located near the Galactic Center. It is hard to determine masses for LBVs and Wolf-Rayet stars, as these are in a He-burning phase, where the interior models (and hence the mass-luminosity relationship) for massive stars are quite uncertain, but it is clear that these stars evolved from stars of mass similar to that of the highest mass R136 stars. The mass inferred for the main-sequence star Sk$-67°$ 211 is also like that of the R136 stars, suggesting that the stars in R136 do not have some kind of special origin. Studies of the youngest OB associations and clusters (i.e., young enough so that not even the highest mass stars would have evolved) show that the richer the cluster is in stars, the higher the mass of the highest mass star seen. We now understand that although physics may indeed impose a limit on how massive a star may be, we have not yet encountered this limit in nature.

Determination of Stellar Masses

How do we determine stellar masses? There are two basic ways: (1) direct determination of masses observationally using binaries, and (2) inference of stellar masses using models.

Stellar Binaries

Simple Newtonian mechanics, specifically Kepler's Third Law of Planetary Motion, allow us to directly determine the masses of stars in some binary star systems.

"Double-lined" spectroscopic binaries are stars whose spectra show the signature of two stars. The orbital periods of these systems are usually a few days or months, and the line-of-sight (radial) velocities of each component can be directly measured as the Doppler effect causes the spectral lines of one star to first appear blue-shifted, and then red-shifted relative to the lines' average position. Masses can be determined *directly* if the orbital inclination can also be found via light variations (i.e., eclipsing or ellipsoidal) or by the direct resolution of such systems through techniques such as speckle imaging or long-baseline interferometry. The masses and luminosities determined from the best, well-separated binaries are shown in Figure 1, and we see that the mass-luminosity relationship inferred from such systems is in excellent agreement with that predicted by modern stellar interior models.

Missing from the figure are any high mass stars; many of these systems are in physical contact or are sufficiently close to have undergone some mass transfer. Searches for high mass spectroscopic binaries whose components are cleanly detached are continuing.

For visual binaries, masses can be determined if the period is short enough to be observed and the distance to the system is known. However, the distance needs to be known to high accuracy for the method to yield useful results: a 7% error in the parallax of the system leads to a 20% accuracy in the masses. There are only 14 systems for which good radial-velocity orbits *and* parallaxes are known, and we include these data in Figure 1. New parallax determinations with the *Hipparcos* satellite will provide improved data on many more systems. Since the orbital periods of these visual binaries are tens or even hundreds of years, reliable measurements over a substantial time period are needed for orbit determinations. High resolution imaging studies with *HST* or new ground-based techniques are providing important new fundamental data on such systems.

Stellar Models

If the effective temperature and luminosity of a star are known (from spectroscopic observations of a star whose distance is known, either from parallax or from membership in a cluster with a known distance), then stellar *interior* models can be used to approximate the star's mass. This method is the basis for most of the masses inferred in determining the initial mass function (discussed in the next section).

It is also possible to estimate a star's mass from stellar *atmosphere* models. Again, spectroscopy is needed of a star with known distance. By fitting the stellar lines and comparison to model atmospheres, it is possible to determine the effective temperature T_{eff} and surface gravity g. Since the star's luminosity L is also known, it is possible to determine the stellar radius R since $L \sim R^2 \times T_{eff}^4$. The mass of the star can then be found since $g \sim M/R^2$.

For the most massive stars, there appears to be a significant "mass discrepancy" between the masses derived from stellar atmospheres and stellar interiors models, stellar atmospheres predicting masses which are systematically smaller. The reason for this discrepancy is unknown at present, but is largest for the most luminous, massive supergiants, for which there may be factors of 2 differences between the two methods. Attempts to resolve this discrepancy by means of spectroscopic binaries have been frustrated by the same lack of identified high mass "detached" systems described above.

Distribution of Stellar Masses

If we were to count stars as a function of mass in the solar neighborhood, we would find that there were far more low mass stars than high mass stars. The reasons for this are basically two-fold. Low-mass stars live much longer than do high mass stars, and so have accumulated over most of the life of the Galaxy, while high mass stars quickly consume their fuel and die. The second reason, however, is that in a typical star-forming event many more low mass stars are formed than are high mass ones.

Knowing the distribution of stellar masses that is obtained when stars form from clouds of gas and dust in space is important for two reasons. First, because the light observed from star clusters and galaxies is dominated by a few of the brightest stars (the tip), it is important to know how many low mass stars are associated with the iceberg as a whole. Indeed in a stellar system such as our own Milky Way galaxy, most of the observed luminosity comes from stars greater than 10 $M_\odot$, while most of the mass is locked up in stars with masses below 1 $M_\odot$. Secondly, the distribution of stellar masses at birth (the *initial mass function*, or IMF) provides clues into the processes of stellar formation. Changes in the shape of this distribution function with mass provide evidence for the critical scales associated with the star formation process.

In general the IMF can be thought of simply as a probability function $\phi(M)$, representing the likelihood of forming a star with a mass between M and $M + dM$. In 1955 Salpeter found that the IMF of stars near the sun was well-represented as a power–law, with $\phi(M) \sim M^{-2.35}$. Modern estimates from studies of OB associations in the Milky Way and LMC suggest that for stars of mass greater than 5 $M_\odot$ the IMF is in-

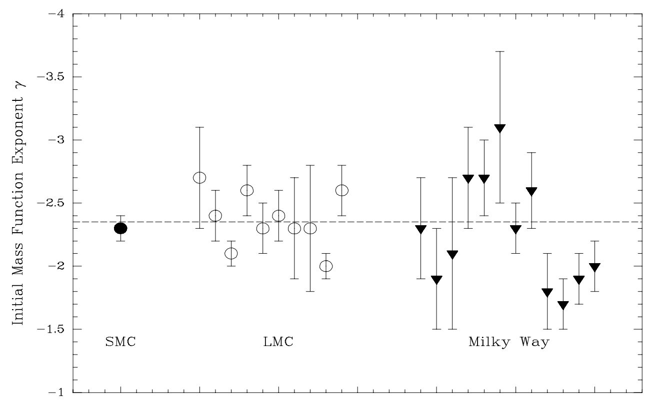

Figure 2: The universality of the initial mass function for high mass stars is demonstrated in this figure showing the IMF exponent γ (where the distribution function $\phi(M) \sim M^\gamma$) for massive stars in the OB associations of the SMC, LMC, and Milky Way. The dashed line is for a Salpeter exponent of -2.35. The metallicities change by a factor of 4 between these three systems.

deed very similar to Salpeter's result, with an exponent of -2.3 ± 0.3 obtaining regardless of cluster density or metallicity (Figure 2).

For intermediate- and low-mass stars, studies of the IMF have traditionally been done using volume-limited samples of stars found in neighborhood of the sun. Using a variety of techniques (photometric, spectroscopic, and parallactic) a luminosity function can be constructed for an unbiased sample of stars. Adopting a mass–luminosity relationship appropriate for the sample in question then allows one to transform the luminosity function into a mass function. In practice, one must take into account the metallicity and evolutionary state of the sample, as well as correct for the relative life-times. The derived IMFs will also depend upon what assumptions have been made about the star-formation of the region, with increased dependence at lower masses, where the star-formation history over the entire life of the galaxy is relevant. Such studies now suggest that the power-law may be somewhat less steep for 1–5 $M_\odot$ stars than for stars of higher mass. For stars of even lower mass, studies are hampered by the additional uncertainty of the mass–luminosity relationship for very cool objects. Most work is consistent with $\phi(M) \sim M^{-1.0\pm0.5}$ over the range 0.1–1 $M_\odot$. There is disagreement, though, as to whether or not the number of low-mass stars that are formed continues to rise to lower and lower masses, or whether the relationship flattens out or even turns over somewhere near the low-mass end of this range.

Another technique that has been exploited in determining the IMF is the use of star clusters in which all stars appear to have roughly the same age. By using such coeval groups of stars, problems inherent in correcting for star-forming histories are eliminated, although other concerns (such as dynamical evolution) need to be addressed. Studies of globular clusters with ages greater than 1 Gyr derive IMFs consistent with the field down to a limiting masses of 0.2–0.3 $M_\odot$. Open clusters with ages 30–500 Myr, perhaps the best place to constrain the IMF near 1 $M_\odot$, confirm that the IMF is less steep than at masses greater than 5 $M_\odot$. Recent studies of nearby young open clusters such as the Pleiades and α Per have begun to probe the IMF down below the hydrogen burning limit. Finally, by studying clusters still embedded in the molecular cloud cores from which stars form, we can attempt to relate different outcomes of the star–forming process with the initial conditions of formation. Thus far, IMFs derived from a wide variety of stellar populations in the Milky Way and local group galaxies are consistent with having been drawn from the same distribution as stars found in the neighborhood of the sun. However, these comparisons are still in their infancy: there could be significant, more subtle variations present in the IMF that have gone undetected.

We have not yet touched at all upon the physical *causes* of the IMF: when an interstellar cloud of dust and gas collapses, what processes dominate and result in the distribution of masses we would observe at some future time? One might naively expect that the dominant physics could be readily deduced from the observed IMF, but this turns out not to be the case, primarily because the IMF appears to be so featureless. Instead, the observed IMF can only be used to constrain star-formation theories at present. Power-law distributions may result from a variety of different scenarios, including so-called "fragmentation" theories. The earliest of these was proposed in 1953 when Hoyle suggested that the *Jeans mass* (the minimum mass needed for gravitational collapse) could result in a hierarchical distribution of masses. The Jeans mass depends upon both temperature and density, and as a cloud collapses the density will increase, while radiation from newly formed stars controls the gas temperature. However, it is not clear whether this elegant model would apply in a real molecular cloud. Alternatively, the agglomeration of protostellar clumps has been suggested as a way to produce a power law. Yet another theory involves feedback from the formation process itself: ignition of a powerful outflow from the protostar might halt further accretion once a characteristic mass has been reached. The resulting distribution of stellar masses would then depend upon the ranges of initial values of a variety of physical parameters. A crucial component in evaluating current theories of star-formation is whether or not the IMF is "scale free" (such would be the case if it were well described by a single power law), or if there is a characteristic mass scale, as is suggested by current observational evidence that the IMF begins to flatten out around 1 $M_\odot$. Solid observational knowledge of the shape of the IMF at even lower masses, and under differing physical conditions, is a prerequisite to understanding star formation.

Bibliography

The following is a list of interesting papers organized in the same order as the topics discussed in this article.

In terms of understanding the physics of stars, and how other stellar parameters depend upon stellar masses, we can do no better than to recommend the following three classics:

Schwarzschild, M 1958 *Structure and Evolution of the Stars* (New York: Dover Publications, Inc.)
Cox J P and Giuli, R T 1968 *Principles of Stellar Structure* (New York: Gordon and Breach)
Hansen C J and Kawaler S D 1994 *Stellar Interiors* (New York: Springer-Verlag).

The realization of these principles can be best seen in actual stellar evolutionary tracks. The dependence of physical parameters on stellar masses used in this article were computed using the evolutionary interior models of Andre Maeder's group in Geneva; these models do an excellent job of matching observations, and are well described in the literature. The most recent version of these models can be found in the following two papers:

Schaller G, Schaerer D, Meynet G, and Maeder A 1992 New grids of stellar models from 0.8 to 120 solar masses at Z=0.020 and Z=0.001 *Astronomy & Astrophysics Supplements* **96**, 269-331
Schaerer D, Meynet G, Maeder A, and Schaller G 1993, Grids of stellar models. II - From 0.8 to 120 solar masses at Z=0.008 *Astronomy & Astrophysics Supplements* **98**, 523-527.

As for the range of stellar masses, the best discussion for the lowest mass stars can be found in:

Burrows A, Hubbard W B, Saumon D, and Lunine J I 1993 An expanded set of brown dwarf and very low mass star models *Astrophysical Journal* **406**, 158-171.

For a modern discussion of the importance of the pulsational instability that may exist for the highest mass stars, we recommend the following:

Appenzeller I 1987 Theory of vibrational instabilities in luminous early type stars In: Lamers, H J G L M and de Loore, C W H (eds) *Instabilities in Luminous Early Type Stars* (Dordrecht: Reidel) pp 68-72.

The exciting discovery of the highest mass stars may be of interest to some readers:

Massey P and Hunter D A 1998, Star formation in R136: a cluster of O3 stars Revealed by Hubble Space Telescope Spectroscopy *Astrophysical Journal* **493** 180-194.

An excellent review of the empirical determination of stellar masses in binary systems is given by:

Popper D M 1980 Stellar masses *Annual Review of Astronomy and Astrophysics* **18** 115-164.

The first reconsideration of visual binaries based upon *Hipparcos* results is given by:

Soderhjelm S Visual Binary orbits and masses post Hipparcos, *Astronomy & Astrophysics*, **341**, 121-140.

A discussion of the "mass discrepancy" for massive stars can be found in:

Burkholder V, Massey P, and Morrell N 1997, The "mass discrepancy" for massive stars: tests of models using spectroscopic binaries" *Astrophysical Journal* **490**, 328-342.

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Scalo, J M 1986 The stellar initial mass function *Fundamentals of Cosmic Physics* 11, 1-278

Kroupa, P, Tout, C A, and Gilmore, G 1993, The distribution of low–mass stars in the Galactic disc *Monthly Notices Royal Astronomical Society*, **262**, 545-587

Meyer, M R Adams, F C, Hillenbrand, L A, Carpenter, J M , and Larson, R B 2000, The stellar initial mass function: constraints from young clusters and theoretical perspectives In: Mannings V G, Boss A, and Russel, S (eds), *Protostars and Planets IV*, (Tucson: The University of Arizona Press), in press

and, in particular, a recent conference on the IMF:

Gilmore, G and Howell, D (eds), *The Initial Mass Function*, (San Francisco, ASP).

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