Light and magnitudes

Ast 401/Phy 580
Fall 2015
Electromagnetic Spectrum

Penetrates Earth's Atmosphere?

Radiation Type
Wavelength (m)

Approximate Scale
of Wavelength

Frequency (Hz)

Temperature of objects at which this radiation is the most intense wavelength emitted

<table>
<thead>
<tr>
<th>Radiation Type</th>
<th>Wavelength (m)</th>
<th>Approximate Scale of Wavelength</th>
<th>Frequency (Hz)</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radio</td>
<td>10^3</td>
<td>Buildings</td>
<td>10^4</td>
<td>1 K (−272 °C)</td>
</tr>
<tr>
<td>Microwave</td>
<td>10^-2</td>
<td>Humans</td>
<td>10^8</td>
<td>100 K (−173 °C)</td>
</tr>
<tr>
<td>Infrared</td>
<td>10^-5</td>
<td>Butterflies</td>
<td>10^12</td>
<td>10,000 K (9,727 °C)</td>
</tr>
<tr>
<td>Visible</td>
<td>0.5×10^-6</td>
<td>Needle Point Protozoans</td>
<td>10^15</td>
<td>10,000,000 K (9,727 °C)</td>
</tr>
<tr>
<td>Ultraviolet</td>
<td>10^-8</td>
<td>Molecules</td>
<td>10^16</td>
<td>10,000,000 K (9,727 °C)</td>
</tr>
<tr>
<td>X-ray</td>
<td>10^-10</td>
<td>Atoms</td>
<td>10^18</td>
<td>10,000,000 K (9,727 °C)</td>
</tr>
<tr>
<td>Gamma ray</td>
<td>10^-12</td>
<td>Atomic Nuclei</td>
<td>10^20</td>
<td>10,000,000 K (9,727 °C)</td>
</tr>
</tbody>
</table>
Optical

Optical observations are limited to about 3200Å-10000Å. The latter is 1 micron (μm)

(About the units: 1Å = 10^{-10} m. It’s about the size of an atom; i.e., the Bohr radius is about 1/2 Å. It’s not unusual to express optical wavelengths in nanometers (10^{-9} m), so we could also say optical is 320nm-1000nm, or 320nm-1μm.)

“Visible” wavelengths (what your eye can see) is about 4,500Å to 6500Å.
Flux

Photometry is the science (art?) of measuring the amount of light coming from an astronomical object, the flux in some bandpass.
Photometry vs Spectrophotometry

With broad- or narrow-band photometry you are measuring the flux integrated over some filter bandpass. Poor wavelength resolution generally, but can be very accurate: 1% is typical “goal” for all sky-photometry; differential photometry can be as good as 0.1% or better.
Light Curve for exoplanet transit

GJ3470 Transit

Light Curve Observed and created by Webber, Barman, Taylor, Levine, Strosahl
Photometry vs Spectrophotometry

Poor spectral resolution though: typical broadband filter is about 500Å wide!
Photometry vs Spectrophotometry

By contrast, spectrophotometry typically has a resolution of about Å, but precision is limited to 1–2%.
Photometry vs Spectrophotometry

So why not always do spectrophotometry?

Conventional photometry has the advantage that you can go much fainter, and more accurate.

Spectrophotometry has the advantage of good wavelength resolution.
Fluxes

Units of flux are usually:

\[ [f_\lambda] = \text{ergs s}^{-1} \text{ cm}^{-2} \text{ Å}^{-1} \text{ or } [f_\nu] = \text{ergs cm}^{-2} \text{ Hz}^{-1} \]

and when we do spectrophotometry, that’s what we measure.

But when we do filter photometry, we are actually measuring a flux averaged over a bandpass:

It’s the flux weighted by the sensitivity function:

\[
F = \frac{\int f_\lambda \times S_\lambda \, d\lambda}{\int S_\lambda \, d\lambda}
\]
Magnitudes

Often what we want to know is the RATIO of fluxes: a star is so much brighter at THIS wavelength than at THAT wavelength, or THIS star is so much brighter than THAT star at the same wavelengths. Magnitudes provide a short-hand way of doing so.
Magnitudes

Consider two stars with fluxes $F_1$ and $F_2$. Then the magnitude difference between them are:

$$m_1 - m_2 = -2.5 \log (F_1/F_2)$$

e.g., if $F_1/F_2 = 100$, then $m_1 - m_2 = -5.0$ magnitudes.

Historically, “magnitudes” came about because the Greeks (Hipparchus or maybe Ptolemy) assigned numbers to the brightness of stars. Really bright stars were 0 or 1, while the faintest you could see was about 6th. Your eye turns out to be logarithmic (surprise!) and 1-->6 was close to 100x.
Magnitudes

Mathematical convenience: what if we wanted to know the flux of a star to 1%? How much is that in magnitudes? Consider two stars, one of which has a flux that is 99% of the other. Then, the difference in magnitudes between them is

\[ m_1 - m_2 = -2.5 \log (f_1) + 2.5 \log (f_2) \]

with \( f_2 = 0.99 f_1 \)

\[ m_1 - m_2 = -2.5 \log (f_1) + 2.5 \log (0.99 f_1) = -2.5 \log (f_1) + 2.5 \log (f_1) + 2.5 \log (0.99) = 0.0109 \text{ mags.} \]

In other words, 1% is about 0.01 mag.
Magnitudes

This trick works well at 10%:
\[ 2.5 \log (0.90) = 0.11 \text{ mag} \]

At 30% it's failing a bit:
\[ 2.5 \log (0.70) = 0.39, \text{ not exactly 0.30.} \]

But at <10% it's a good approximation!
Magnitudes

Vega has a magnitude of 0. It was decided to simply DEFINE the magnitude scale at each wavelength relative to Vega. (Well, more or less...)

So, when we say a star has a magnitude of 13.25 in the “B” band we are really saying that the star is $10^{-(13.25)/2.5}$ x as bright as Vega is in the B band. Whatever that is.
Magnitudes

Or to put it another way, if a star has a flux that is 0.01 \times that of Vega, then it has a magnitude of 5.0.
### Magnitudes

<table>
<thead>
<tr>
<th>Object</th>
<th>Magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>-26.7</td>
</tr>
<tr>
<td>Full Moon</td>
<td>-12</td>
</tr>
<tr>
<td>Sirius</td>
<td>-1.4</td>
</tr>
<tr>
<td>Vega</td>
<td>+0.0</td>
</tr>
<tr>
<td>Betelgeuse</td>
<td>+0.4</td>
</tr>
<tr>
<td>Polaris</td>
<td>+2.0</td>
</tr>
<tr>
<td>Naked Eye Limit</td>
<td>+6</td>
</tr>
<tr>
<td>Visual limit 6-inch</td>
<td>+13.0</td>
</tr>
<tr>
<td>CCD 5min 6-inch</td>
<td>+16.0</td>
</tr>
<tr>
<td>Hubble Deep Field</td>
<td>+30.0</td>
</tr>
</tbody>
</table>
An image from the Hubble Space Telescope reveals two equally bright stars, each with $V=14.00$, separated by just 0.1”. If you observe these from the ground, where their light will be combined, how bright will the (composite) star appear to be?
Let $f_1$ and $f_2$ be the flux from each of the two stars. (Note $f_1 = f_2$.)

$V_1 = -2.5 \log(f_1) + C = 14.00 \implies f_1 = f_2 = 10^{-\frac{(14-C)}{2.5}}$

$f_1 + f_2 = 2 \times 10^{-\frac{(14-C)}{2.5}}$

$V_{combined} = -2.5 \log(f_1 + f_2) + C =
-2.5 \log(2 \times 10^{-\frac{(14-C)}{2.5}}) + C =
-2.5 \log(2) + -2.5 \times (\frac{(14-C)}{2.5}) + C =
-2.5 \log(2) + 14 + C - C = 13.25$
Colors

We use filters to limit the wavelength range when we do photometry. Most common is UBVRI in the optical, and JHK in the near-infrared.
B-V = -2.5 \log \left( \frac{F_B}{F_V} \right) + \text{Const}

What’s with this “+ Const”? Recall that the magnitude system is set up relative to Vega, essentially a 10,000° K black-body. What we really have in terms of fluxes then is

B-V = -2.5 \log \left( \frac{F_B}{F_{B,Vega}} / \frac{F_V}{F_{V,Vega}} \right). \text{ But } F_{B,Vega} \neq F_{V,Vega}! \text{ So, we just have to remember that a black-body with a temperature of } 10,000° \text{ K is defined to have zero colors in the UBVRI system.}
Colors

Because bigger magnitudes mean fainter, a star with a $B-V = -0.3$ is very, very blue. One with a $B-V = +2.0$ is very, very red. All normal stars are between these extremes.

Other fun colors: $U-B$, $B-V$, $V-R$, $R-I$, and including near infra-red colors, $J-K$ and a wonderful combination, $V-K$. Stars have different ranges among these.
How do we measure a magnitude?

When we observe at the telescope, we actually obtain the number of ADUs per second for a star. This is the INSTRUMENTAL magnitude. We get the magnitude then by comparing this to a standard star. There are some complications, and we’ll be talking a lot about that in the next few weeks.
Imagine I observe a star with the BLT and measure a count rate of 100 ADUs/sec in V, and 200 ADUs/sec in B. The star is:

a) Red
b) Blue
c) Don’t know
Imagine I observe a star with the BLT and measure a count rate of 100 ADUs/sec in $V$, and 200 ADUs/sec in $B$. The star is:

a) Red

b) Blue

c) Don’t know
Okay, now imagine you observe a standard star at the same airmass of the previous star, that has a B−V of 0.0 and a V magnitude of 12.0. It has a count rate of 1000 ADUs/sec through the V filter and 2000 ADUs/sec through the B filter. What is the brightness at V of the aforementioned star (V: 100 ADUs/sec and B: 200 ADUs/sec):

a) 14.5  
c) 17.0  

b) 12.0  
d) 9.5  
e) Still can’t tell
Okay, now imagine you observe a standard star at the same airmass of the previous star, that has a $B-V$ of 0.0 and a $V$ magnitude of 12.0. It has a count rate of 1000 ADUs/sec through the V filter and 2000 ADUs/sec through the B filter. What is the brightness at $V$ of the aforementioned star ($V$: 100 ADUs/sec and B: 200 ADUs/sec): 

a) 14.5  c) 17.0  
b) 12.0  d) 9.5  e) Still can’t tell
Doing photometry...

Okay, now imagine you observe a standard star at the same airmass of the previous star, that has a B-V of 0.0 and a V magnitude of 12.0. It has a count rate of 1000 ADUs/sec through the V filter and 2000 ADUs/sec through the B filter. What is the color (B-V) of the aforementioned star? (V: 100 ADUs/sec and B: 200 ADUs/sec):

a) 0.75 \((-2.5 \log (2))\)                     c) -0.75

b) 0.00                                      d) Can’t tell
Okay, now imagine you observe a standard star at the same airmass of the previous star, that has a B–V of 0.0 and a V magnitude of 12.0. It has a count rate of 1000 ADUs/sec through the V filter and 2000 ADUs/sec through the B filter. What is the color (B–V) of the aforementioned star? (V: 100 ADUs/sec and B: 200 ADUs/sec):

a) 0.75 \((-2.5 \log (2))\)  
c) -0.75  
b) 0.00  
d) Can’t tell
Quick Summary

* Astronomers like to use magnitudes to characterize fluxes because magnitudes are logarithmic in nature, and we usually are only interested in the RELATIVE brightness of things.

\[ m_\lambda = -2.5 \log (f_\lambda) + C_\lambda \]

* If star A is 10x brighter than star B at a particular wavelength, its magnitude is 2.5 magnitudes smaller:

\[
\begin{align*}
    m_{\lambda A} &= -2.5 \log (f_{\lambda A}) + C_\lambda \\
m_{\lambda B} &= -2.5 \log (f_{\lambda B}) + C_\lambda \\
\rightarrow m_{\lambda A} - m_{\lambda B} &= -2.5 \log (f_{\lambda A}) + C_\lambda - [-2.5 \log (f_{\lambda B}) + C_\lambda ] \\
    &= -2.5 \log (f_{\lambda A}/f_{\lambda B}) \\
    &= -2.5 \log(10.) = -2.5
\end{align*}
\]
Absolute magnitudes

What we’ve been talking about so far have all been APPARENT magnitudes—how bright objects appear to us on Earth in a particular band-pass.

For comparing objects, what we really need to know is the ABSOLUTE magnitude—what magnitude the object would have if it were at some standard distance, 10 parsecs.
Absolute magnitudes

Since flux varies by $r^{-2}$ we can expect

$$\frac{f_{\text{observed}}}{f_{\text{absolute}}} = \frac{100}{r^2}.$$  

Let $M_V = \text{absolute magnitude}$, and $m_V$ be apparent magnitude, both at V. Then:

$$m_V - M_V = -2.5 \log \left( \frac{f_{\text{observed}}}{f_{\text{absolute}}} \right) = -2.5 \log \left( \frac{100}{r^2} \right) = 5 \log r - 5$$

“$m_V - M_V$” is called the DISTANCE MODULUS.
Absolute magnitudes

Thus \( M_V = m_V - 5 \log (r) + 5 \)

where \( r \) is the distance to the object in parsecs (pc).

Let’s try it and see. \( m_V \) for the sun is -26.7, and it’s at a distance of 1 AU. There are 206265 AUs in a parsec, so \( r = 1/206265 \text{ pc} = 4.84 \times 10^{-6} \text{ pc} \).

So \( M_V(\text{sun}) = -26.74 - 5 \times \log (4.84 \times 10^{-6}) + 5 = 4.83 \).

If the sun were at a distance of 10 parsec, it would be visible, but only faintly!
Absolute magnitudes

In contrast, consider the star Deneb. It has $m_V=1.3$ and is located a distance of 490 pc. What’s its absolute magnitude?

$$1.3-M_v = 5 \log (490) - 5$$

$M_V=-7.2$. If Deneb was this close, it would be 10x brighter than Venus at its brightest ($V=-4.8$).

Compare this to $+4.8$ for the sun! Deneb is intrinsically 12 mags more luminous (at V), or about 70,000x.
Absolute magnitudes

Two stars are intrinsically identical (same size, some effective temperature). One has an apparent magnitude of 10, and the other one had an apparent magnitude of 15. How much further away is the fainter one?

a) 10x
b) 100x
c) 1000x
d) Huh?
Absolute magnitudes

Two stars are intrinsically identical (same size, some effective temperature). One has an apparent magnitude of 10, and the other one had an apparent magnitude of 15. How much further away is the fainter one?

a) 10x
b) 100x
c) 1000x
d) Huh?
Bolometric Magnitudes

Often an observer wants to know the total flux integrated all wavelengths in order to compare to theoretical models. This total flux is sometimes given as the bolometric magnitude, \(M_{\text{bol}}\).

\[M_{\text{bol}} = M_V + \text{B.C.},\]

where the B.C. is the “bolometric correction”, computed from model atmospheres as a function of effective temperature or B-V. The B. C. is not 0 for Vega or for the sun, but is somewhat arbitrarily defined so that the B.C. is never greater than 0.
Here are some bolometric corrections for main-sequence stars according to Kurucz model atmospheres:

<table>
<thead>
<tr>
<th>Teff</th>
<th>B-V</th>
<th>B.C.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3500</td>
<td>1.9</td>
<td>-2</td>
</tr>
<tr>
<td>4000</td>
<td>1.7</td>
<td>-1.1</td>
</tr>
<tr>
<td>5000</td>
<td>0.9</td>
<td>-0.4</td>
</tr>
<tr>
<td>6000</td>
<td>0.6</td>
<td>-0.2</td>
</tr>
<tr>
<td>7000</td>
<td>0.4</td>
<td>-0.1</td>
</tr>
<tr>
<td>8000</td>
<td>0.2</td>
<td>-0.1</td>
</tr>
<tr>
<td>10000</td>
<td>0</td>
<td>-0.4</td>
</tr>
<tr>
<td>25000</td>
<td>-0.2</td>
<td>-2.6</td>
</tr>
<tr>
<td>50000</td>
<td>-0.3</td>
<td>-4.5</td>
</tr>
</tbody>
</table>
Bolometric Magnitudes

$M_{bol}$ for the sun is 4.75. So, if we want the total flux relative to the sun, we have:

$M_{bol} = 4.75 = -2.5 \log \frac{L}{L_{sun}}$

$M_{bol} = -2.5 \log \frac{L}{L_{sun}} + 4.75$
If a star is 100x more (bolometrically) luminous than the sun, what is its bolometric magnitude?

a) $4.75 - 5 = -0.25$

b) $4.75 + 5 = 9.75$

c) Where did 5 come from?
If a star is 100x more (bolometrically) luminous than the sun, what is its bolometric magnitude?

a) $4.75 - 5 = -0.25$

b) $4.75 + 5 = 9.75$

c) Where did 5 come from?
Special flashback reminder!

Stefan-Boltzmann law:

$$L = 4\pi\sigma R^2 T_{\text{eff}}^4$$
An astronomer named Don Hayes and collaborators rationalized this in the 1970s. Using a black-body light source they borrowed from the National Bureau of Standards, they used the same telescope and instrument to measure it as Vega. They mounted the black-body on the catwalk of the 4-meter and observed it from across the mountain with the 2.1-meter telescope! What they found was:

\[ m_\nu = -2.5 \log f_\nu - 48.60 \text{ if } f_\nu \text{ is in ergs cm}^{-2} \text{ Hz}^{-1} \]
Magnitudes

Those magnitudes are sometimes referred to as “AB” magnitudes or “spectrophotometric magnitudes.” They agree with V magnitudes (more or less) but differ at other wavelengths as 0.00 mag corresponds to constant mν.
1) You measure a magnitude of a star as +10.0. But it turns out this star is a binary star, consisting of two stars, one of which is 2x brighter than the other. What would the individual magnitudes of the two components be if you could measure them? Hint: $m_{\text{total}}$ is not equal to $m_1+m_2$. Instead, it's the fluxes that add.

2) This binary star is in a cluster you know is 1000 pc distant. What is the absolute visual magnitude of the two components?