Spherical Triangles!

Ast 401/Phy 580

Fall 2015

“Spherical Astronomy”

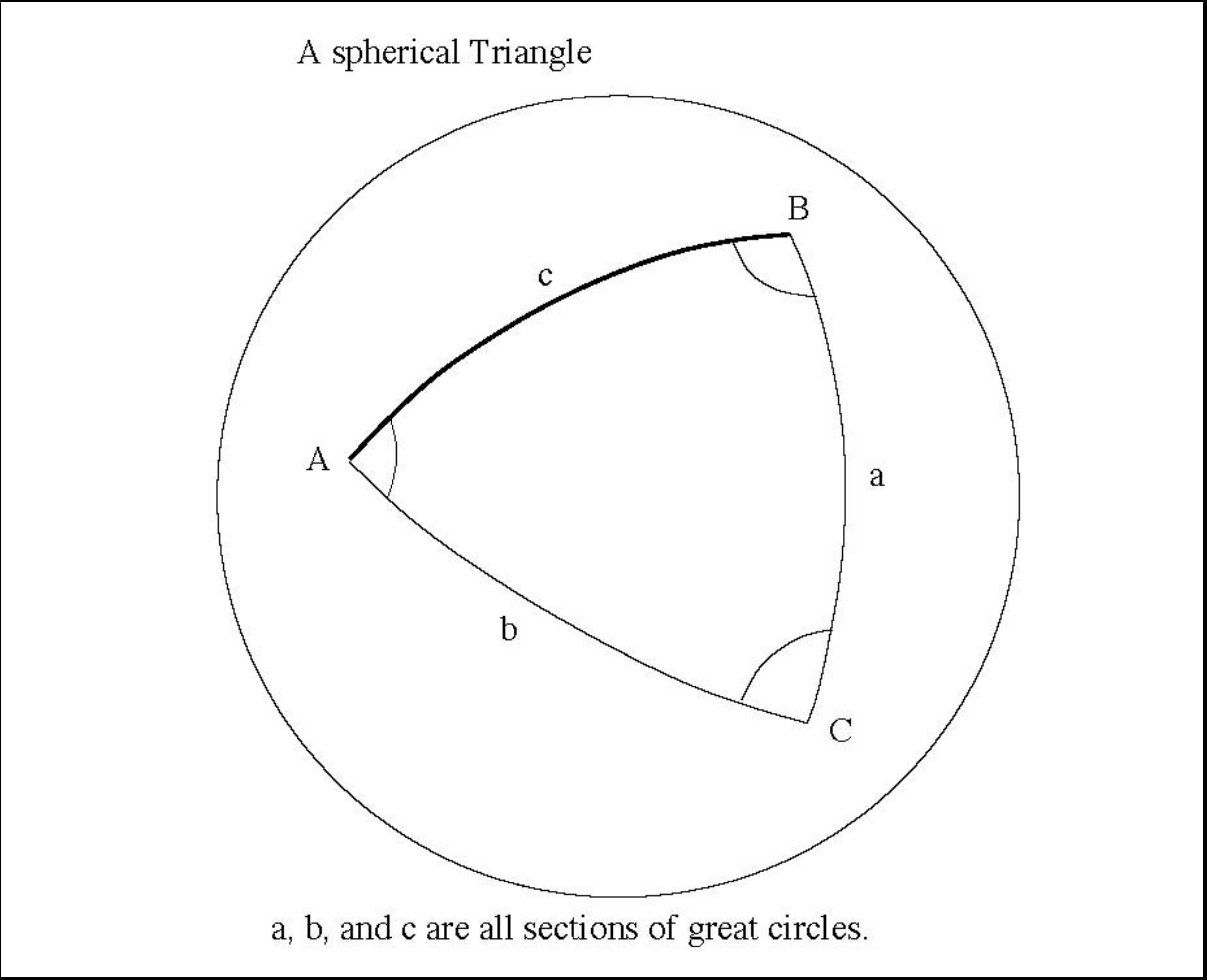
Geometry on the surface of a sphere different than on a flat plane

No straight lines!

Why do we care? Want to handle angles and arcs on the Celestial Sphere we’ve been discussing!

Mastering “spherical trig” will allow us to compute the angular distance between two stars and convert from one coordinate system to another (e.g., equatorial to alt/az).

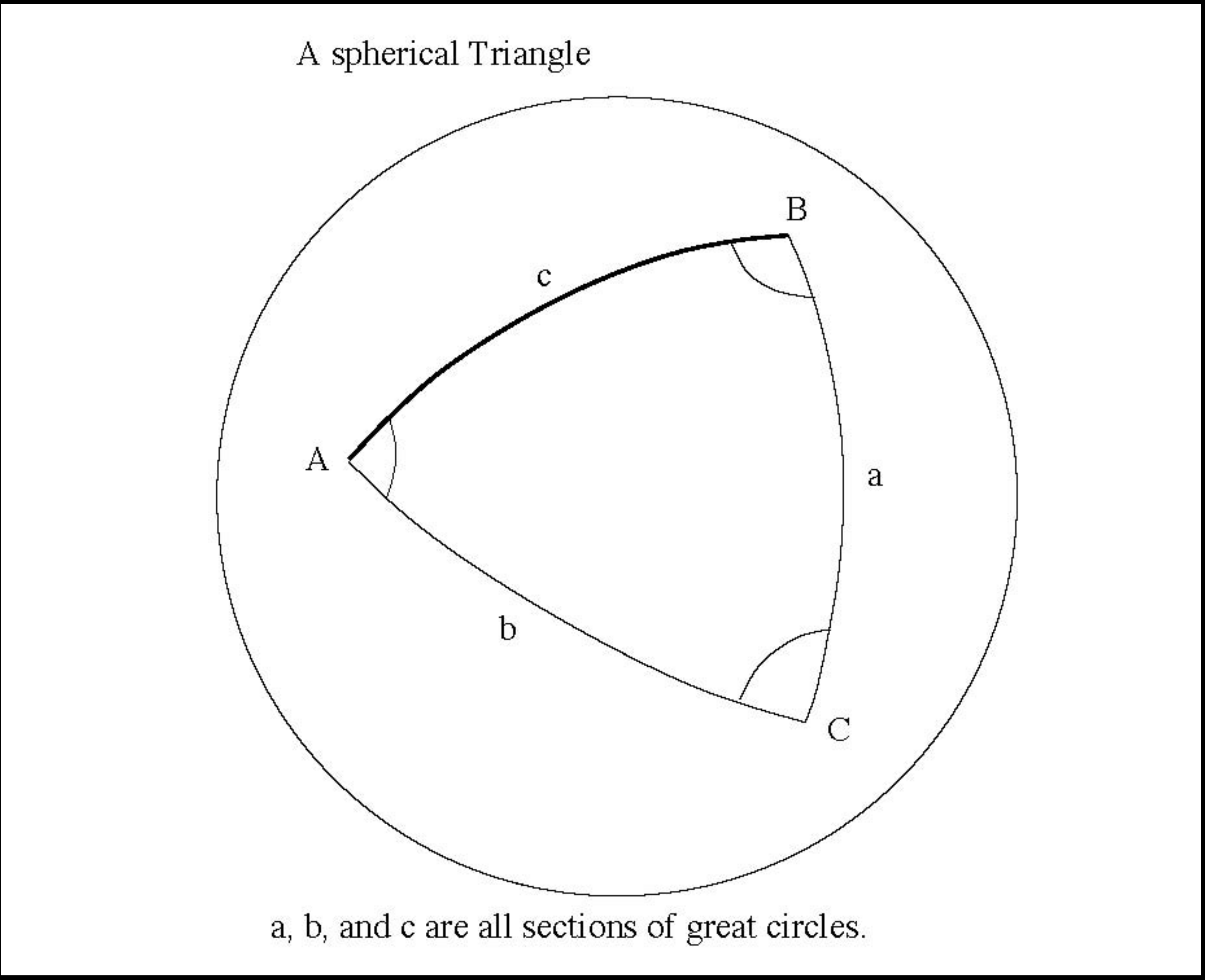
Spherical triangles



A, B, and C are angles, measured in degrees or radians)

a, b, c are lengths of arcs, ALSO measured in degrees or radians

Spherical triangles



A, B, and C will be 0–180° or 0–π

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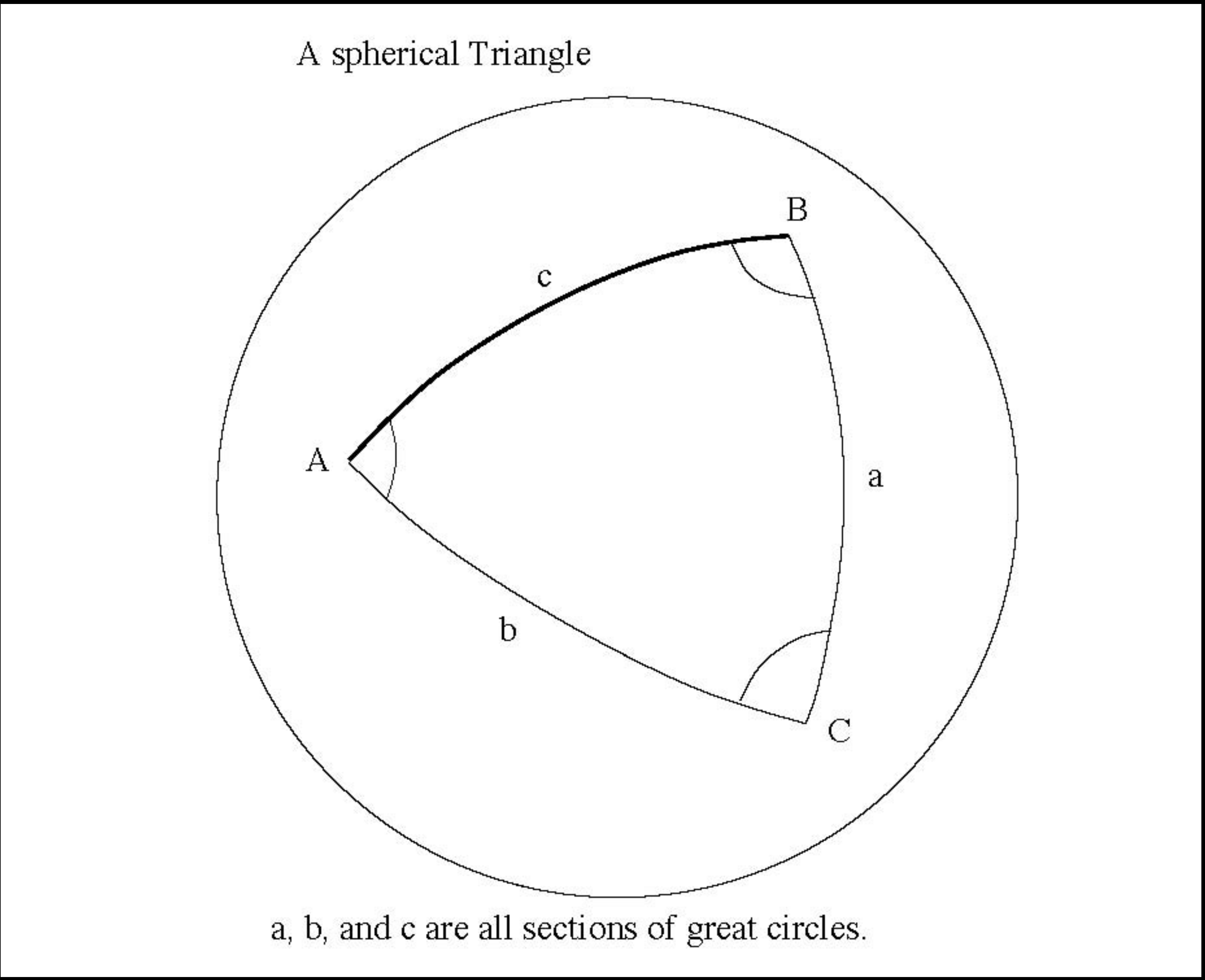
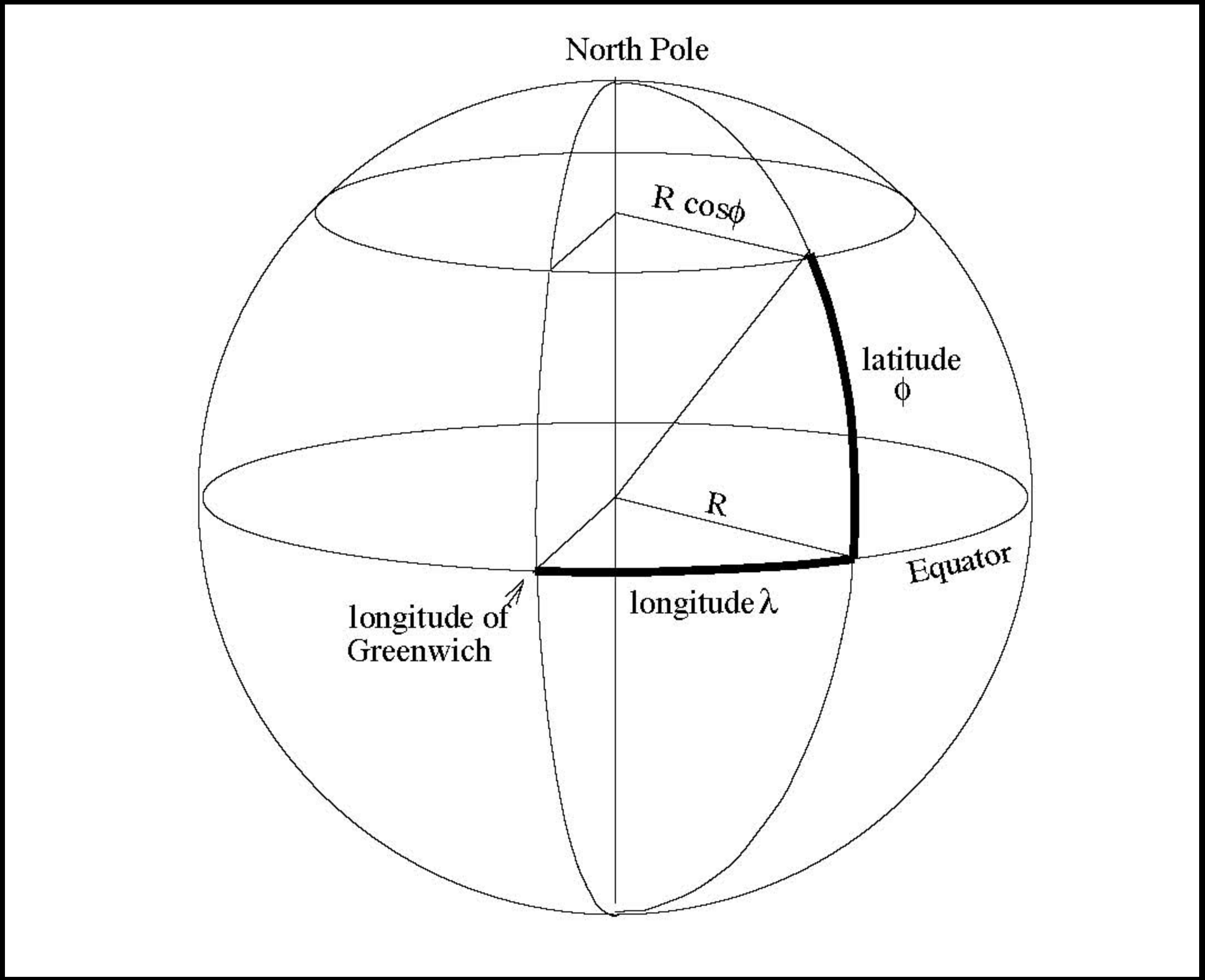
Spherical triangles



In a plane, the angles of an triangle always add up to 180°. In a spherical triangle, they add to ≥ 180° !!!

Great Circles

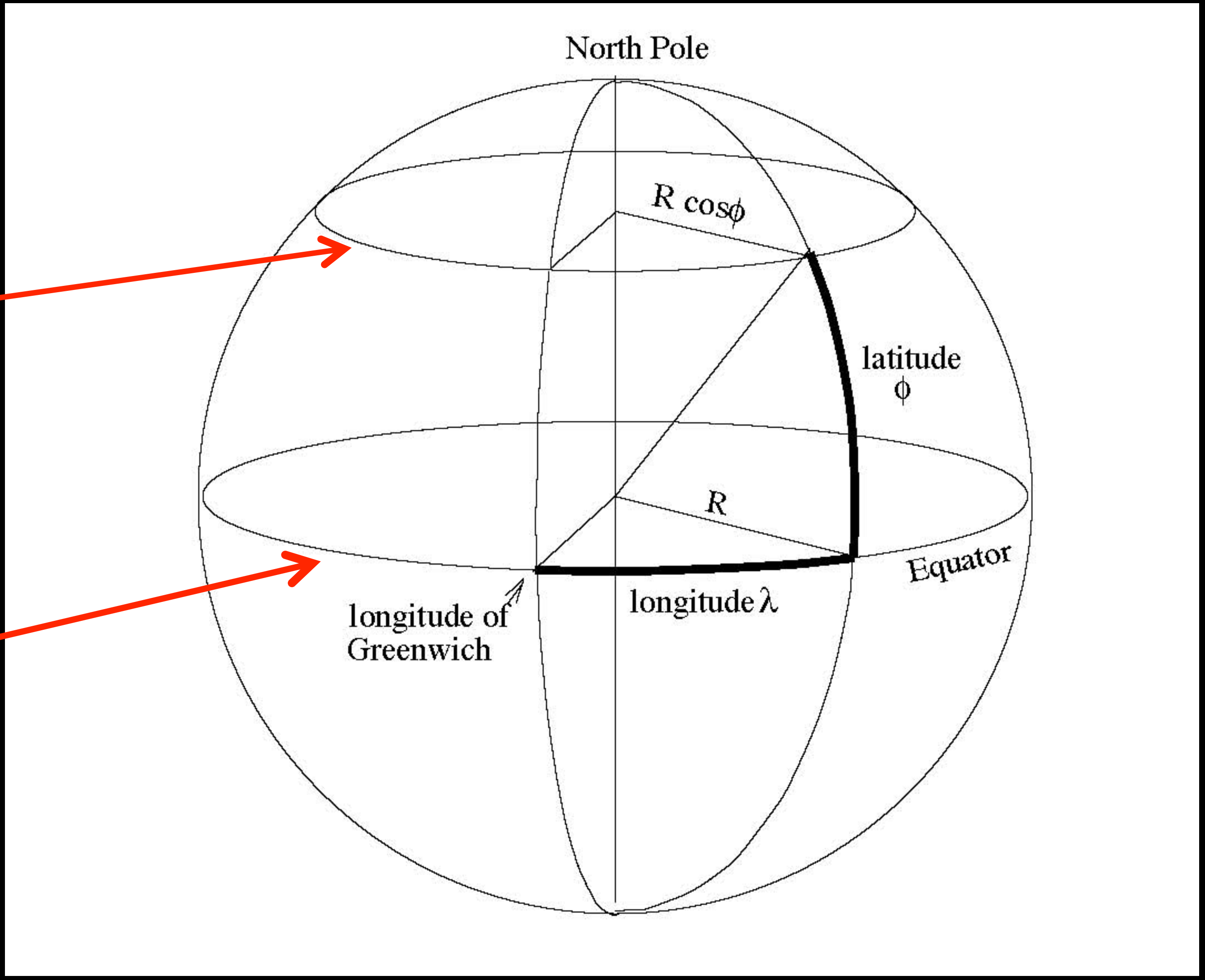
- Intersection of plane through the center of sphere.
- Radius equals the radius of the sphere.
- Any two points on the surface (plus center of sphere) define a unique great circle.
- Shortest distance between two points on the surface of the sphere is a curve that is part of a great circle.
- Airplanes fly along great circles to conserve fuel.



Great and "not-so-great" circles

The circle of constant latitude Φ is NOT a great circle.

The equator IS a great circle.



Important side-bar: what is that in dog-years?

We are used to measuring angles in degrees (°). We can divide a degree up into 60 pieces, called arcminutes ('). We can divide each arcminute up into 60 pieces called arcseconds ("). So, a latitude on earth, or a declination in the sky, might be given as +41°13'52.5". To convert to decimals, it would be 41° + 31'/60. + 52.523"/3600. = 41.53126°.

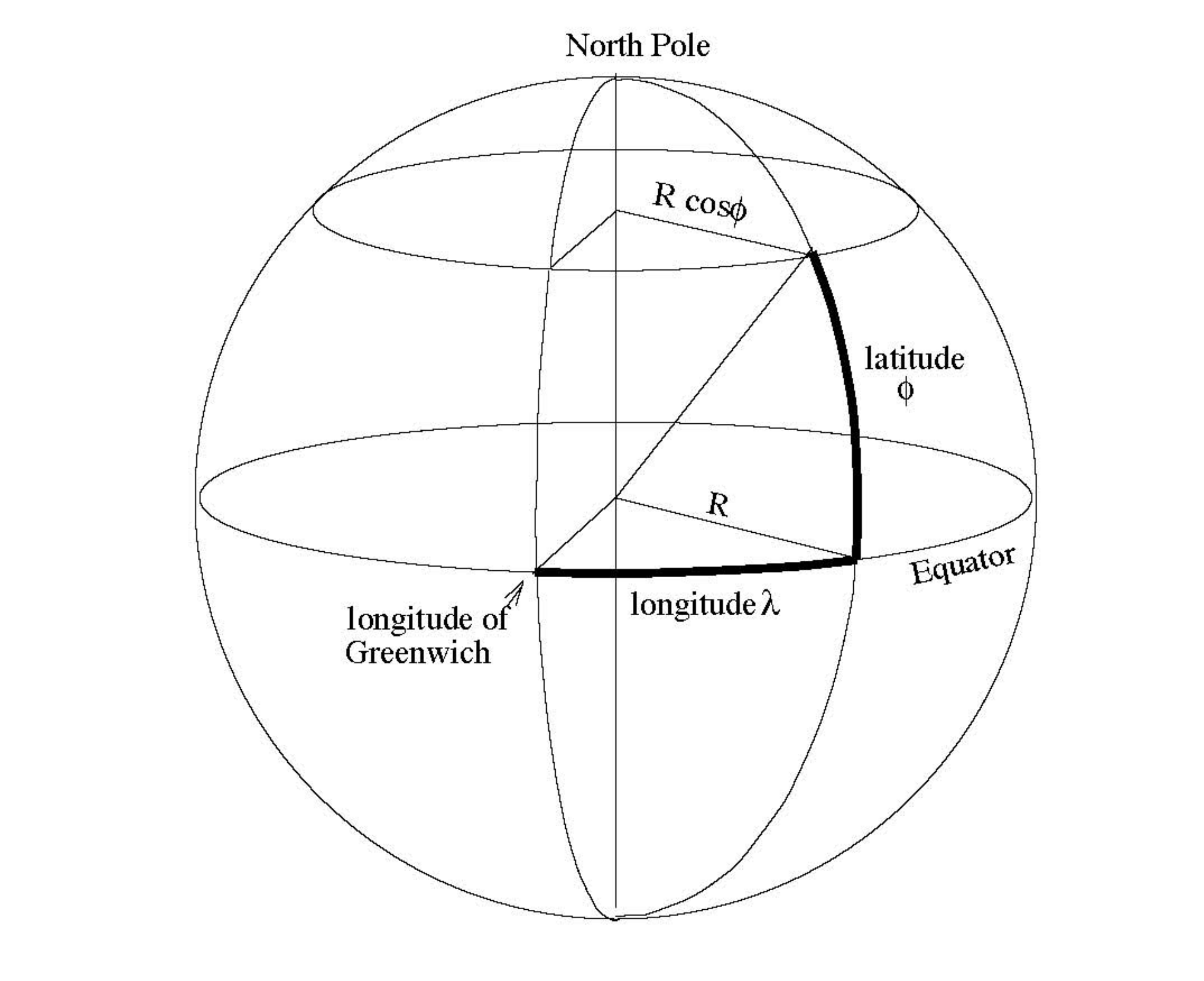
Note that 1" = 0.00028°. So if we care about 0.1" we should report decimal degrees to ≈0.00003°

1" is how much angle a dime subtends of 2.3 miles.

Important side-bar: what is that in dog-years?

Right ascension in the sky is given as hours, minutes, and seconds (longitude will also sometimes be reported this way, east or west of Greenwich). On the equator, 1^s of RA = 15" of arc length. However, as we go north or south, the amount of arc-length decreases!

If you walked around the earth at the equator, it would take you 25,000 miles. But if you walked around the earth at a latitude of 45° it would only take you 17,700 miles. At the north pole you could walk around the earth in a single step (well, none).



In all cases you've moved through an ANGLE of 360°. But the arc lengths differ by the cosine of the latitude (or declination).

The circumference of the Earth is 25,000 miles. If you started walk/swim along a line of constant latitude of +60°, how far would you go before you came back to your starting place?

A. 50,000 miles.

B. 25,000 miles

C. 12,500 miles

D. You'd fall off the Earth before then.

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Two stars in the Large Magellanic Cloud have RAs that differ by 1^{min} but the same declination (-60^o). How far apart are they in arcseconds?

A. $60^s \times 15'' \times \cos(-60^o) = 450''$

B. $60^s \times 15'' / \cos(-60^o) = 1800''$

C. $60^s \times 15'' = 900''$ (there are always 24^{hr} in a circle)

D. Huh?

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Important side-bar: what is that in dog-years?

So, in general, $1^s = 15'' \cos(\text{declination})$. 1^s is simply not worth as much up near near the pole!

This also implies that if you want to give RA and DEC to similar precision, you need an extra decimal place on the RA seconds compared to DEC arcseconds.

Vega:

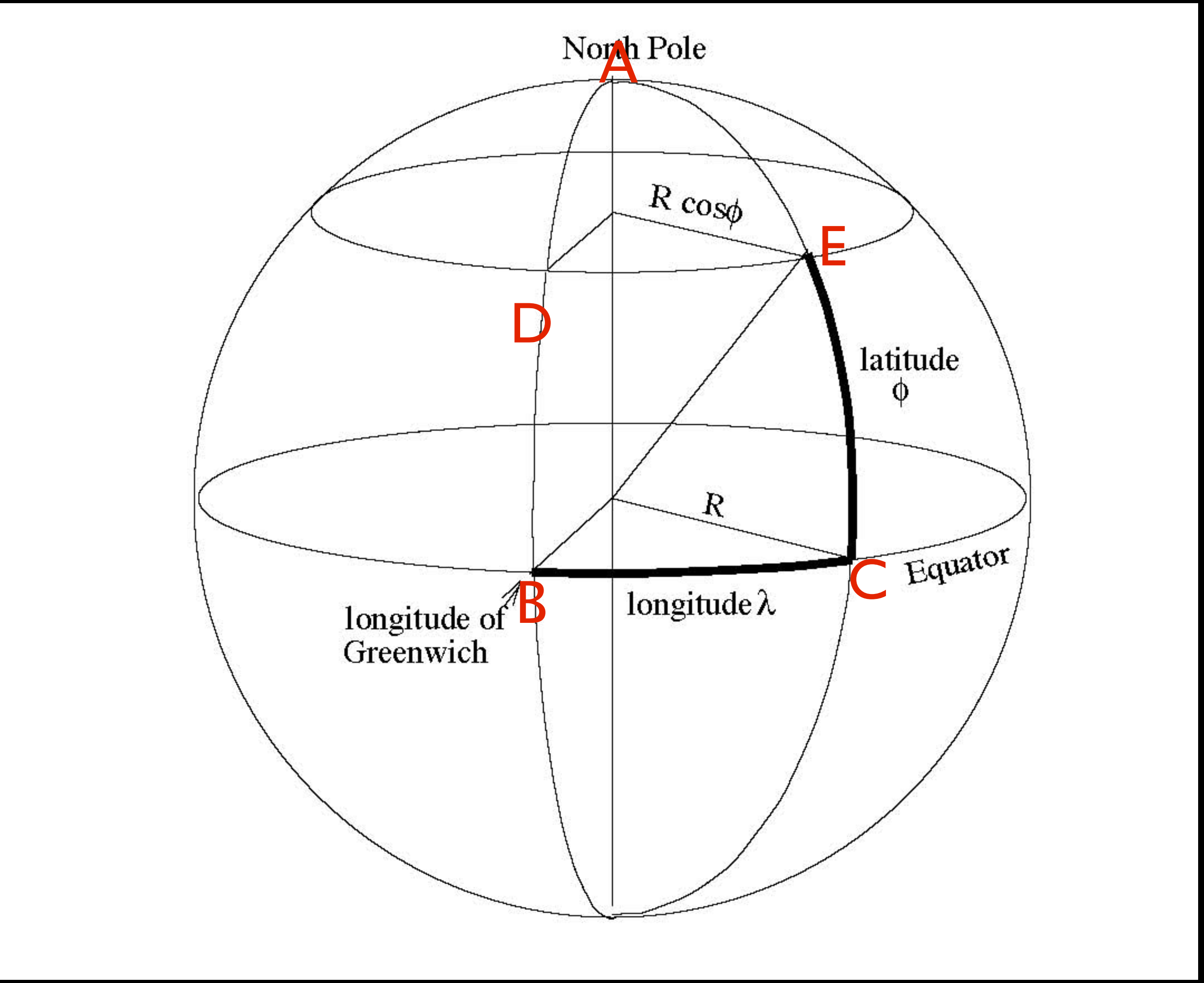
RA=19:36:56.336 DEC=+38°48′01.28″

or

RA=19:36:56.34 DEC=+38°48′01.3″

Great and "not-so-great" circles

Spherical triangles are made up ONLY of arcs of GREAT circles, not small circles.



Any two points and the POLE CAN define a spherical triangle. (ABC)

It's possible to draw a triangle on a sphere with two points and the pole which is not a spherical triangle, e.g., ADE. (But of course you COULD make a spherical triangle connecting those three points by moving the arc DE.)

Basic trig relationships spherical triangles

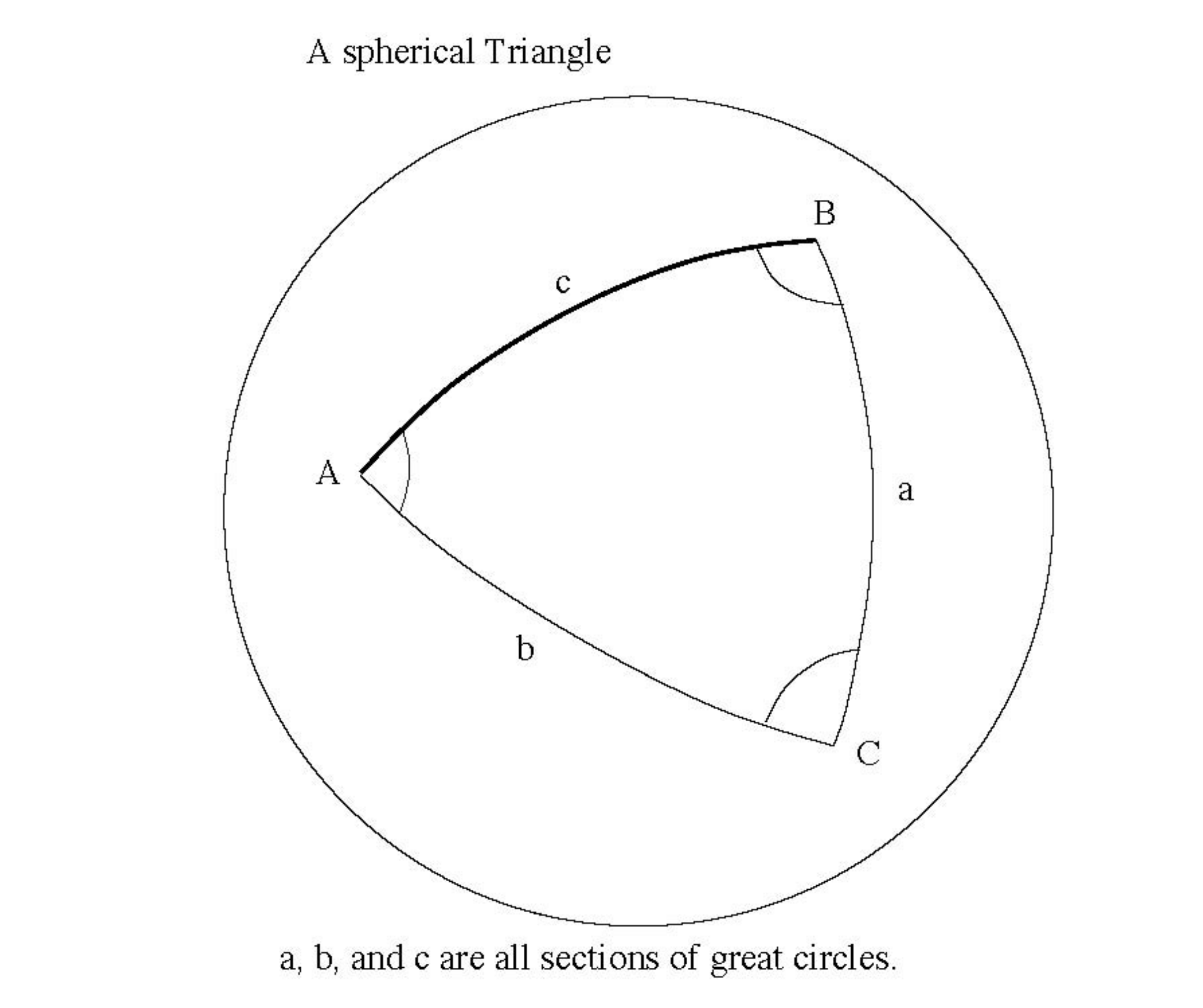
Cosine formula:

$$\cos(a) = \cos(b)\cos(c) + \sin(b)\sin(c)\cos(A)$$

Relates three sides and one angle

Sine formula:

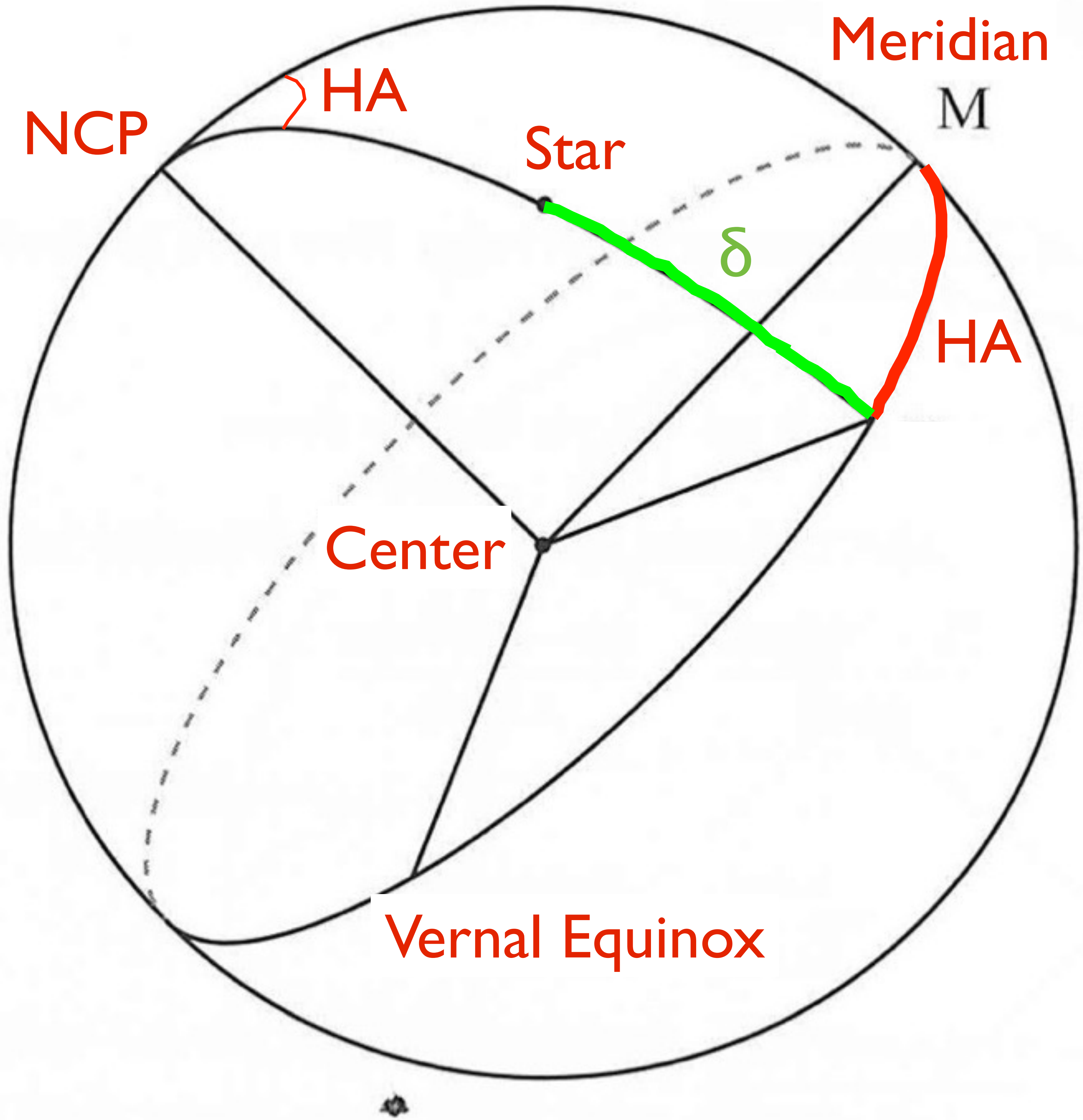
$$\left(\frac{\sin(A)}{\sin(a)}\right) = \left(\frac{\sin(B)}{\sin(b)}\right) = \left(\frac{\sin(C)}{\sin(c)}\right)$$



With these two formula, you can rule the world! Or at least all spherical triangles.

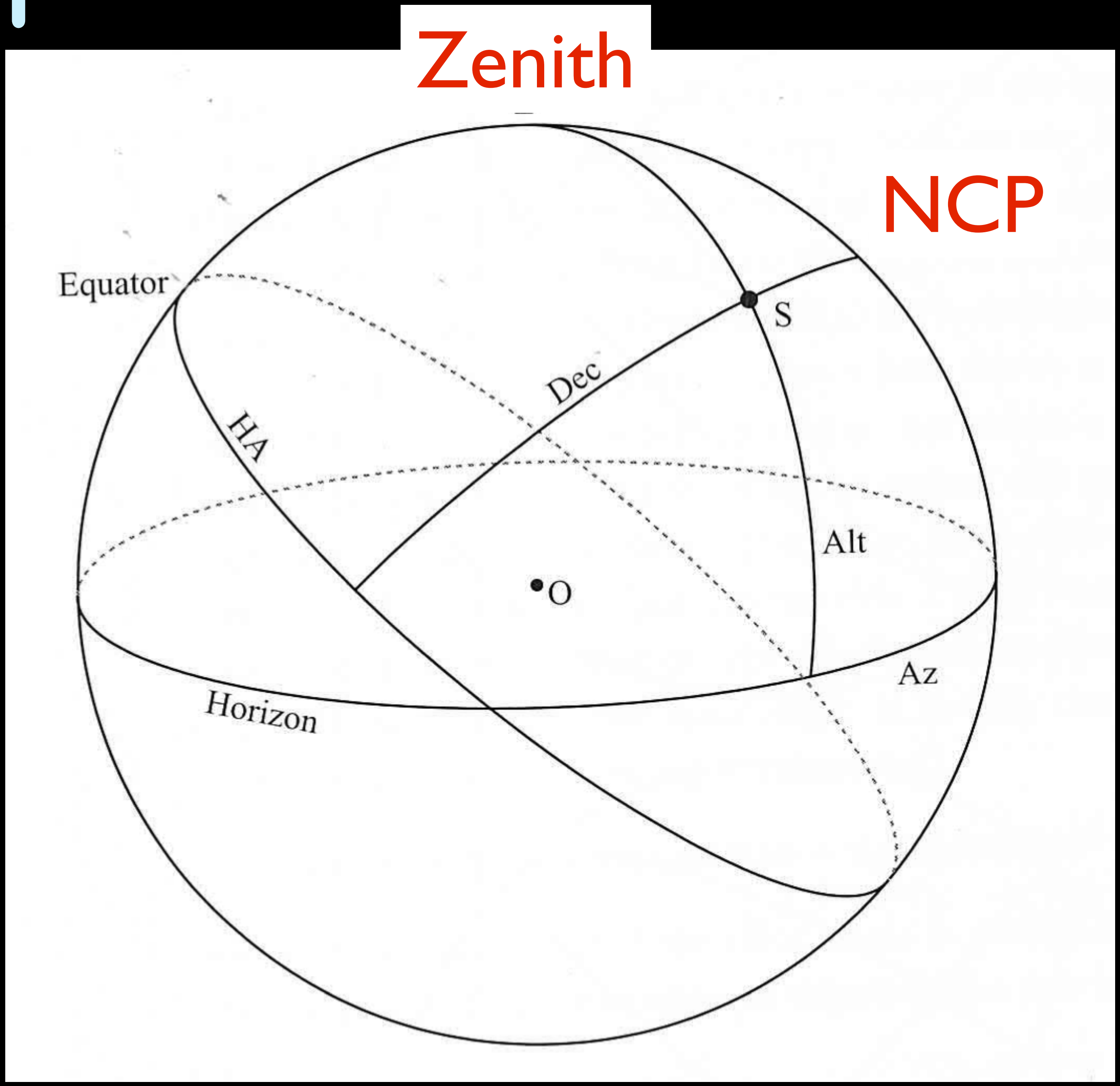
Return to Hour Angle

We need to really understand hour angle if we're going to convert from equatorial to alt/az or back.



Meridian
M
HA
NCP
Star
δ
HA
Center
Vernal Equinox

Converting from equatorial to alt az

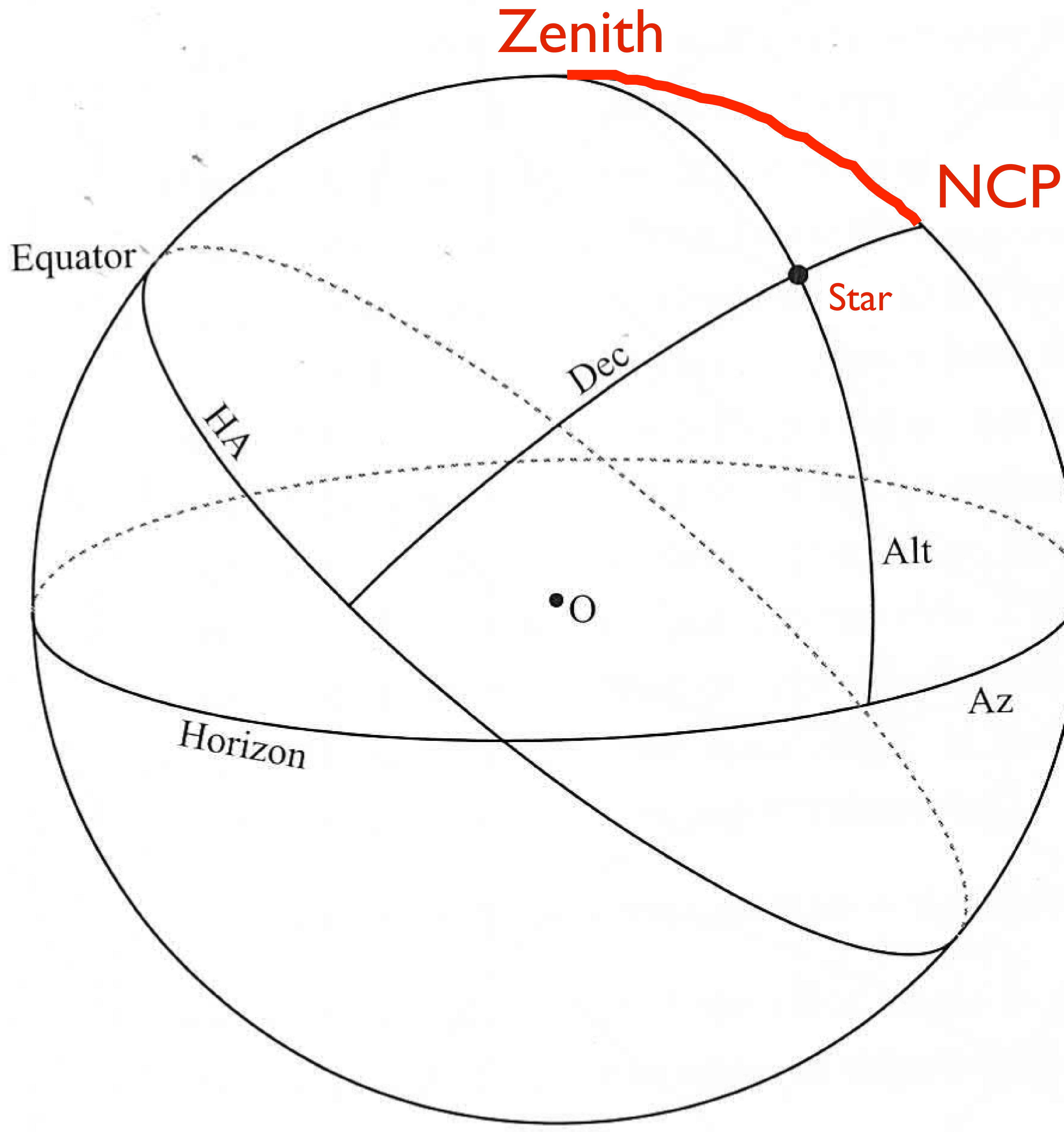


Converting from equatorial to alt az

First rule is to draw a great circle from the POLE of one system to the POLE of the other

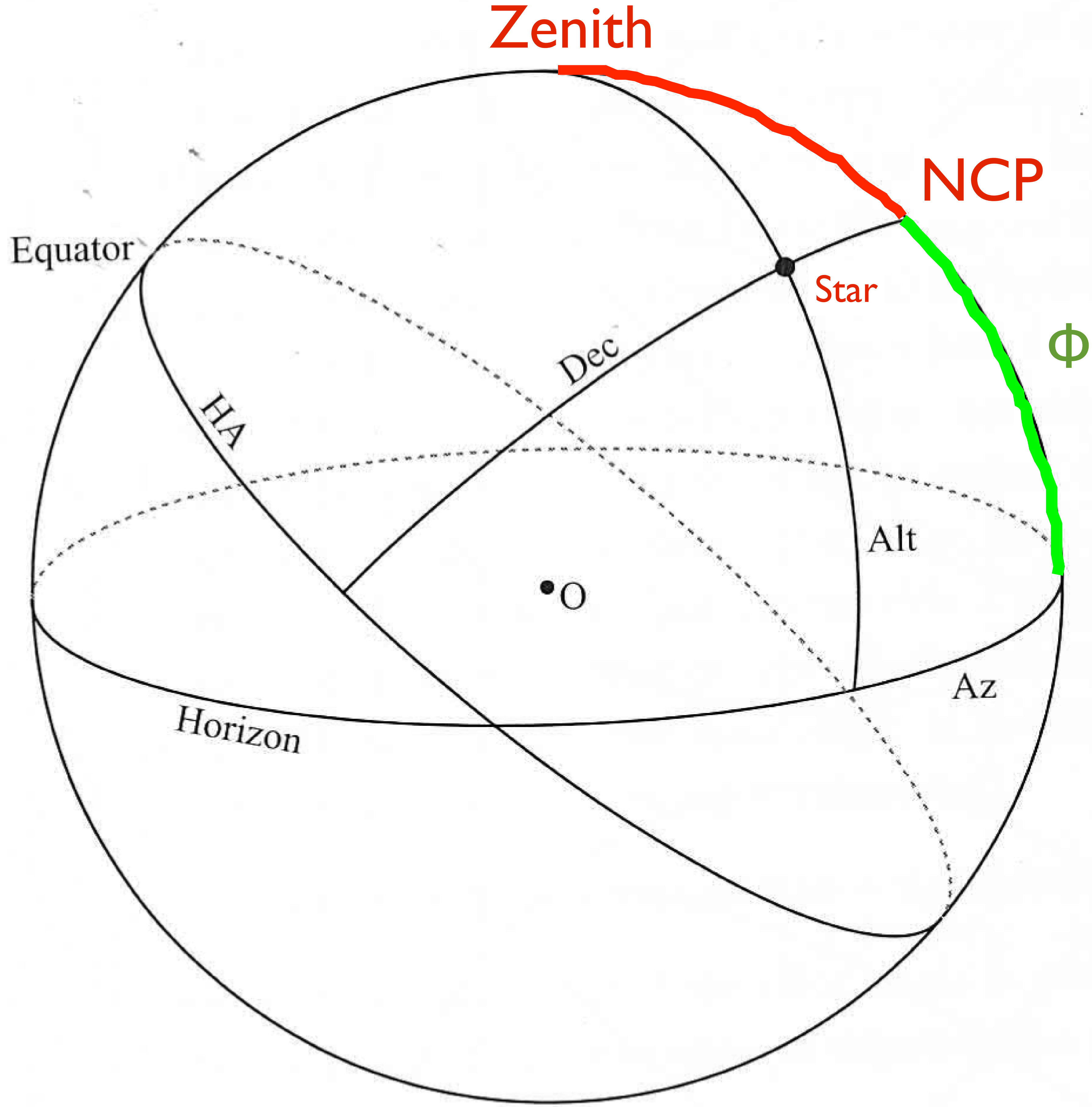
Pole of equatorial: North (or south) celestial pole (NCP, SCP)

Pole of alt/az: Zenith


Zenith
NCP
Equator
Star
Dec
HA
Alt
O
Az
Horizon

Converting from equatorial to alt az

What do we know about that arc?


Zenith
NCP
Φ
Star
Equator
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HA
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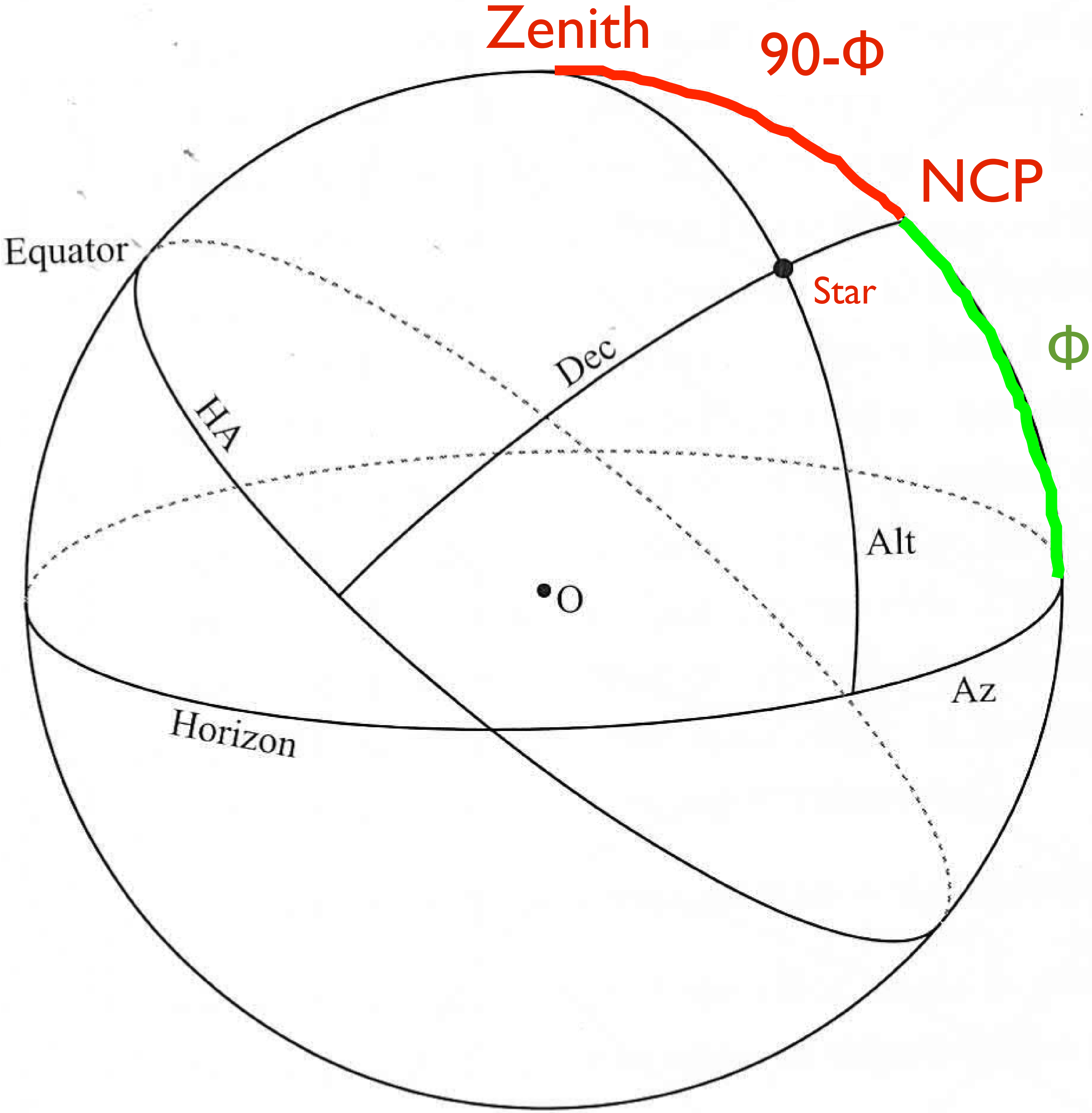
O
Az
Horizon


Zenith
90-Φ
NCP
Equator
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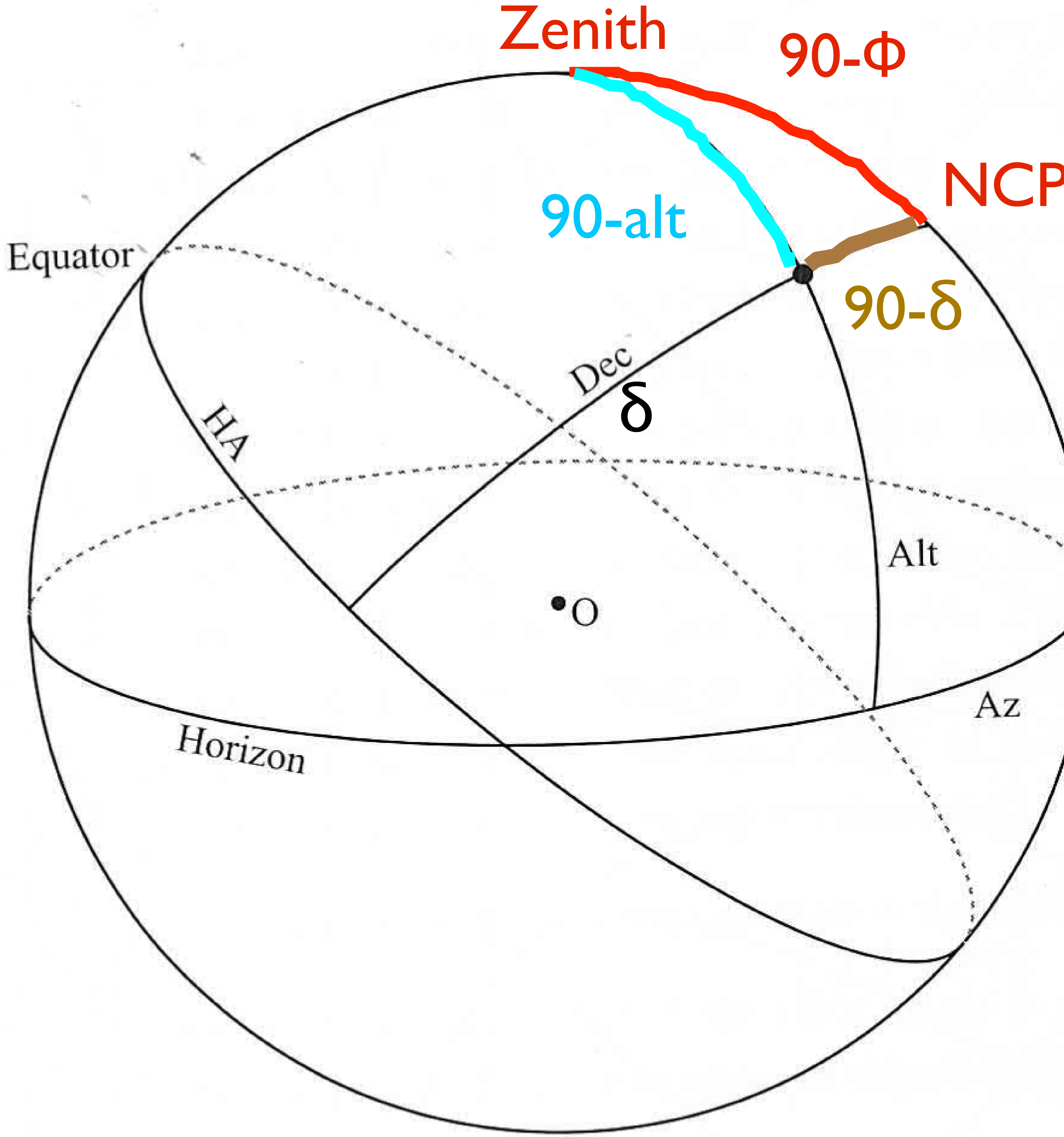
Az
Horizon

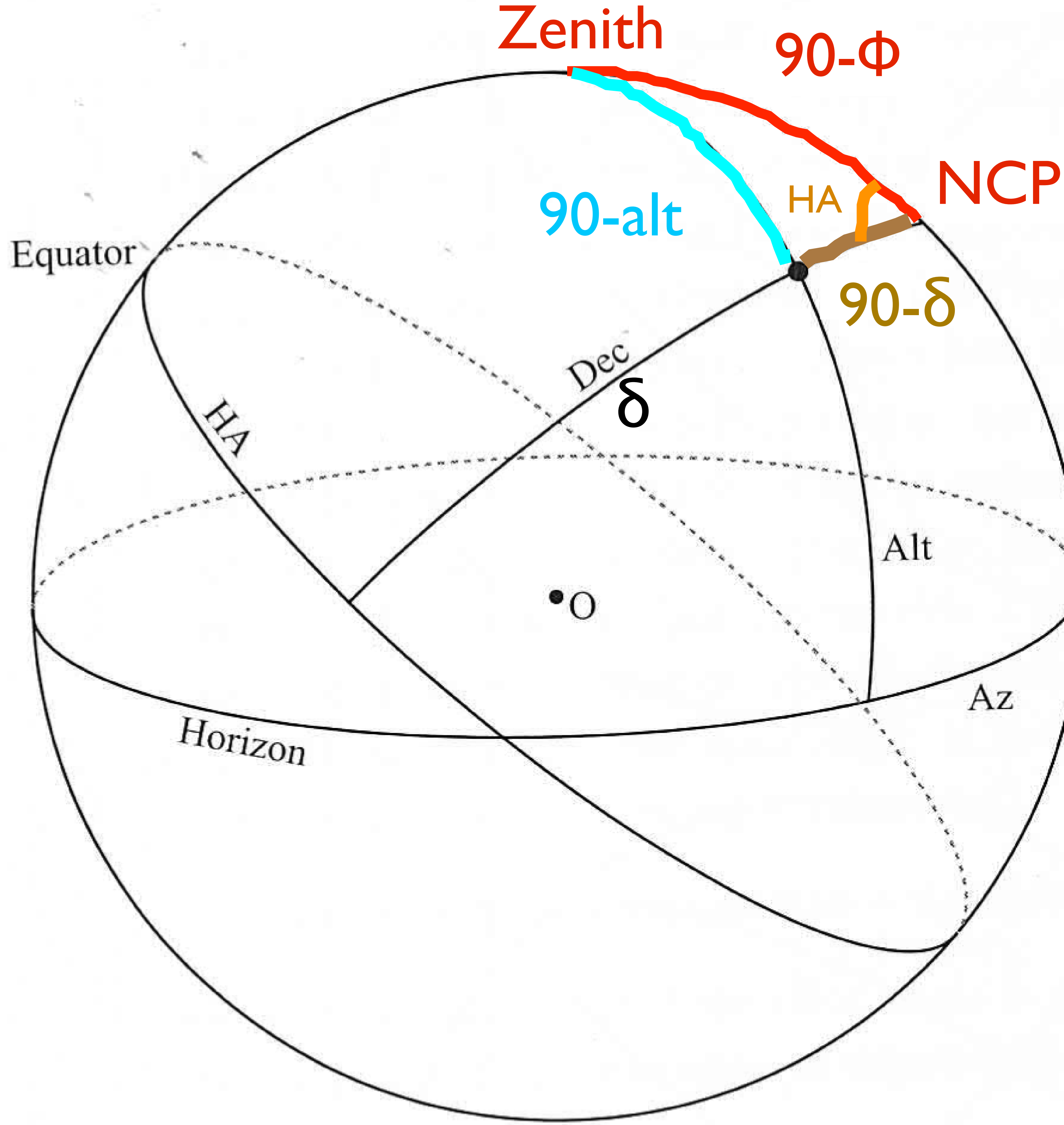
Converting from equatorial to alt az

What about the other sides?


Zenith
90-Φ
NCP
90-alt
Equator
90-δ
Dec
δ
HA
Alt
O
Az
Horizon

Converting from equatorial to alt az

Oooohhhh! And we know all three sides plus one angle!


Zenith
90-Φ
NCP
HA
90-alt
90-δ
Equator
Dec
δ
HA
Alt
O
Az
Horizon

Basic trig relationships spherical triangles

Cosine formula:

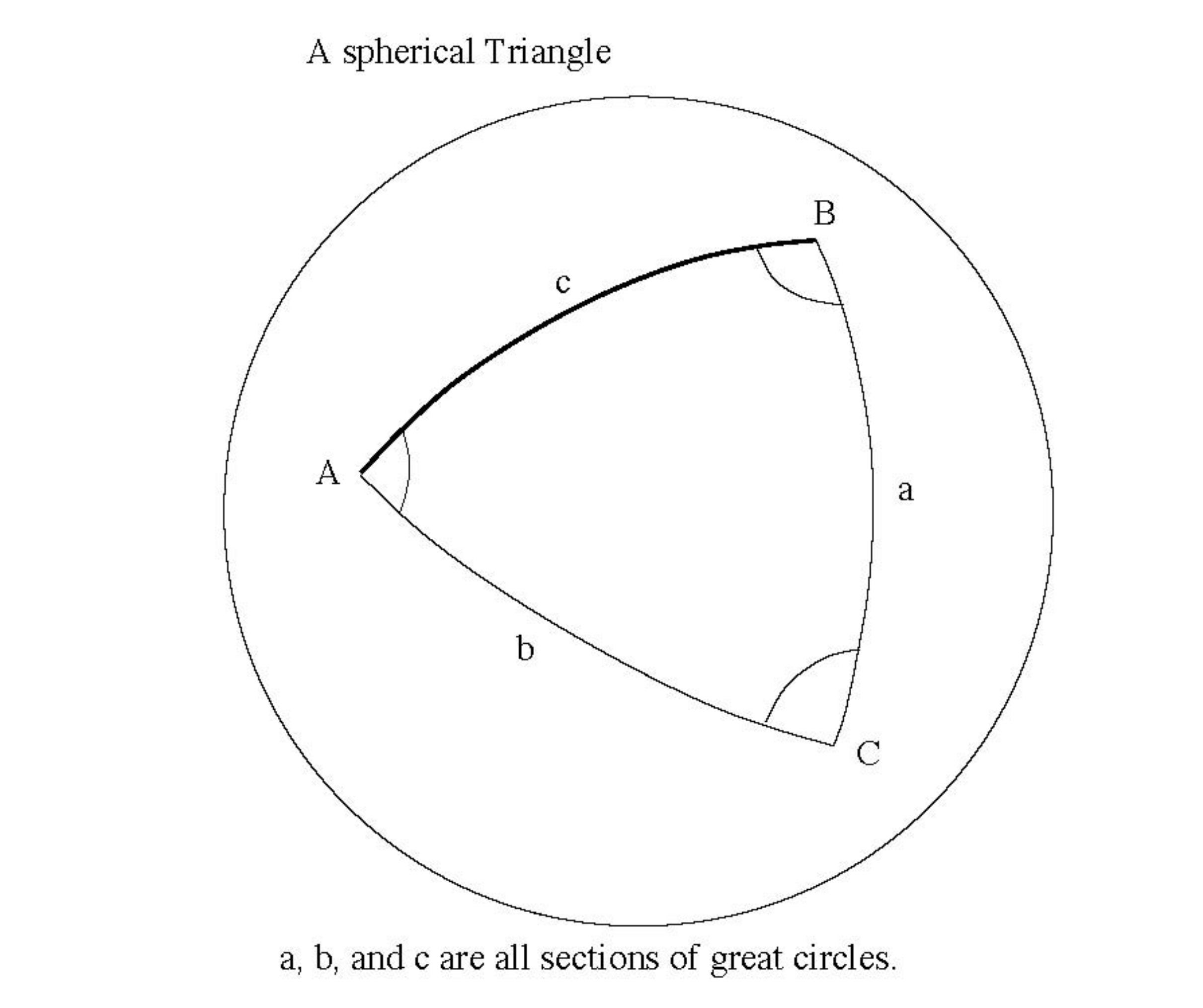
$$\cos(a) = \cos(b)\cos(c) + \sin(b)\sin(c)\cos(A)$$

So A=hour angle=HA

b=90-latitude = 90-Φ

c=90-declination=90-δ

a=90-altitude



$$\cos(90-alt)=\cos(90-\Phi)\cos(90-\delta)+\sin(90-\Phi)\sin(90-\delta)\cos(HA)$$

Converting from equatorial to alt az

$$\cos(90-alt)=\cos(90-\Phi)\cos(90-\delta)+\sin(90-\Phi)\sin(90-\delta)\cos(HA)$$

Reminder:

$$\cos(90-x)=\sin x$$
$$\sin(90-x)=\cos x$$

$$\sin(alt)=\sin(\Phi)\sin(\delta)+\cos(\Phi)\cos(\delta)\cos(HA)$$

Converting from equatorial to alt az

$$\cos(90-\text{alt})=\cos(90-\Phi)\cos(90-\delta)+\sin(90-\Phi)\sin(90-\delta)\cos(HA)$$

Reminder:

$$\cos(90-x)=\sin x$$
$$\sin(90-x)=\cos x$$

$$\sin(\text{alt})=\sin(\Phi)\sin(\delta)+\cos(\Phi)\cos(\delta)\cos(HA)$$

Example:

The Andromeda Galaxy (RA=00^h40^m dec=$+41^o$) is observed from Flagstaff (latitude $\Phi = 35^o\ 11'\ 53''$) on Oct 21 at local midnight. How high up (altitude) will it be?

1. What's the LST at local midnight on Oct 21? It's 00:00 on Sept 21 at midnight, so it must be about 02:00.
2. What's the hour angle? 02:00-00:40 = 01:20. Convert to degrees: 15^o/hr x 1.333hr = 20.0^o

$$\sin(alt)=\sin(\Phi)\sin(\delta)+\cos(\Phi)\cos(\delta)\cos(HA)$$

$$\sin(alt)=\sin(35.2^o)\sin(41^o)+\cos(35.2^o)\cos(41^o)\cos(20^o)$$

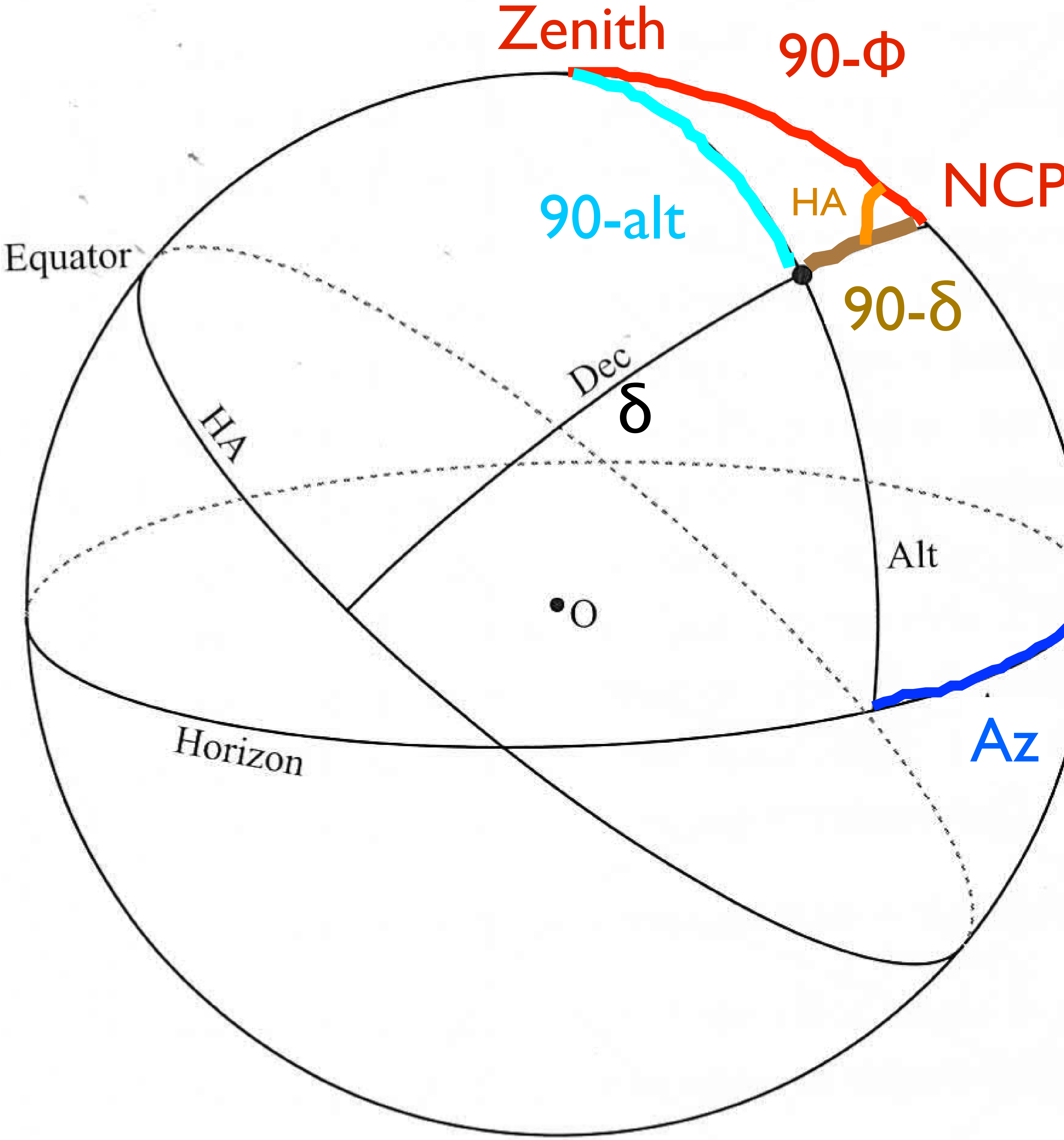
$$\sin(alt)=\sin(35.2^o)\sin(41^o)+\cos(35.2^o)\cos(41^o)\cos(20^o)= 0.958$$

$$altitude=\sin^{-1}(0.958) =73.3^o$$

(Insert rant here about significant digits).

Converting from equatorial to alt az

What about the Azimuth?



Zenith
90-Φ
NCP
HA
90-alt
90-δ
Equator
Dec
δ
HA
Alt
O
Horizon
Az

Converting from equatorial to alt az

What about the Azimuth?

From the law of sines:

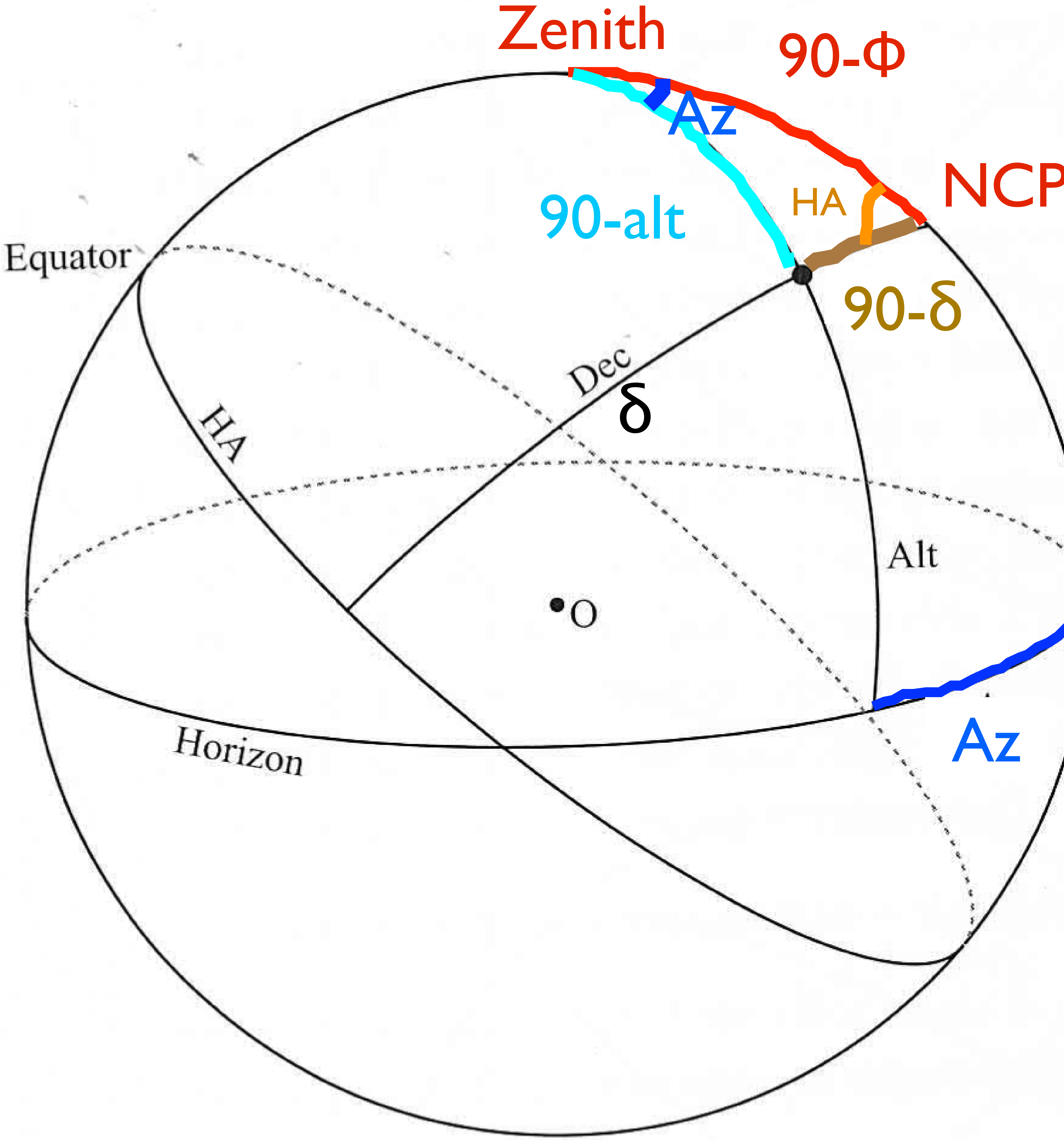
$$\sin(Az)/\sin(90^o-\delta) = \sin(HA)/\sin(90^o-alt)$$

$$\sin(Az)=\cos(\delta)\sin(HA)/\cos(alt)=$$

$$\cos(41^o)\sin(20^o)/\cos(73.3^o)= 0.898$$

$$Az=\arcsin(0.898)=63.9^o$$

But wait! That's not reasonable!! The star is in the WEST. Az has to be >180°



Zenith
90-Φ
Az
90-alt
HA
NCP
Equator
90-δ
Dec
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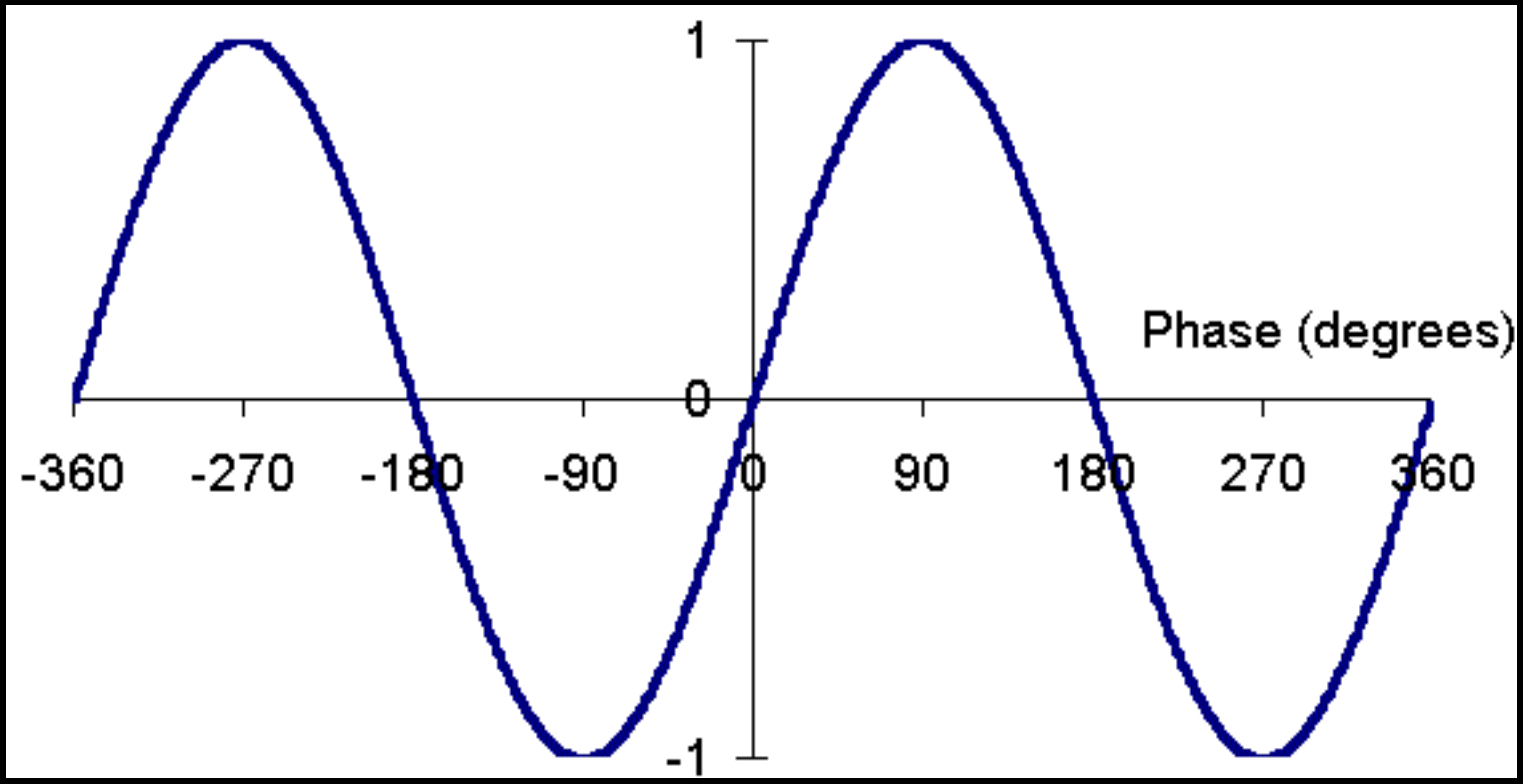
O
Horizon
Az

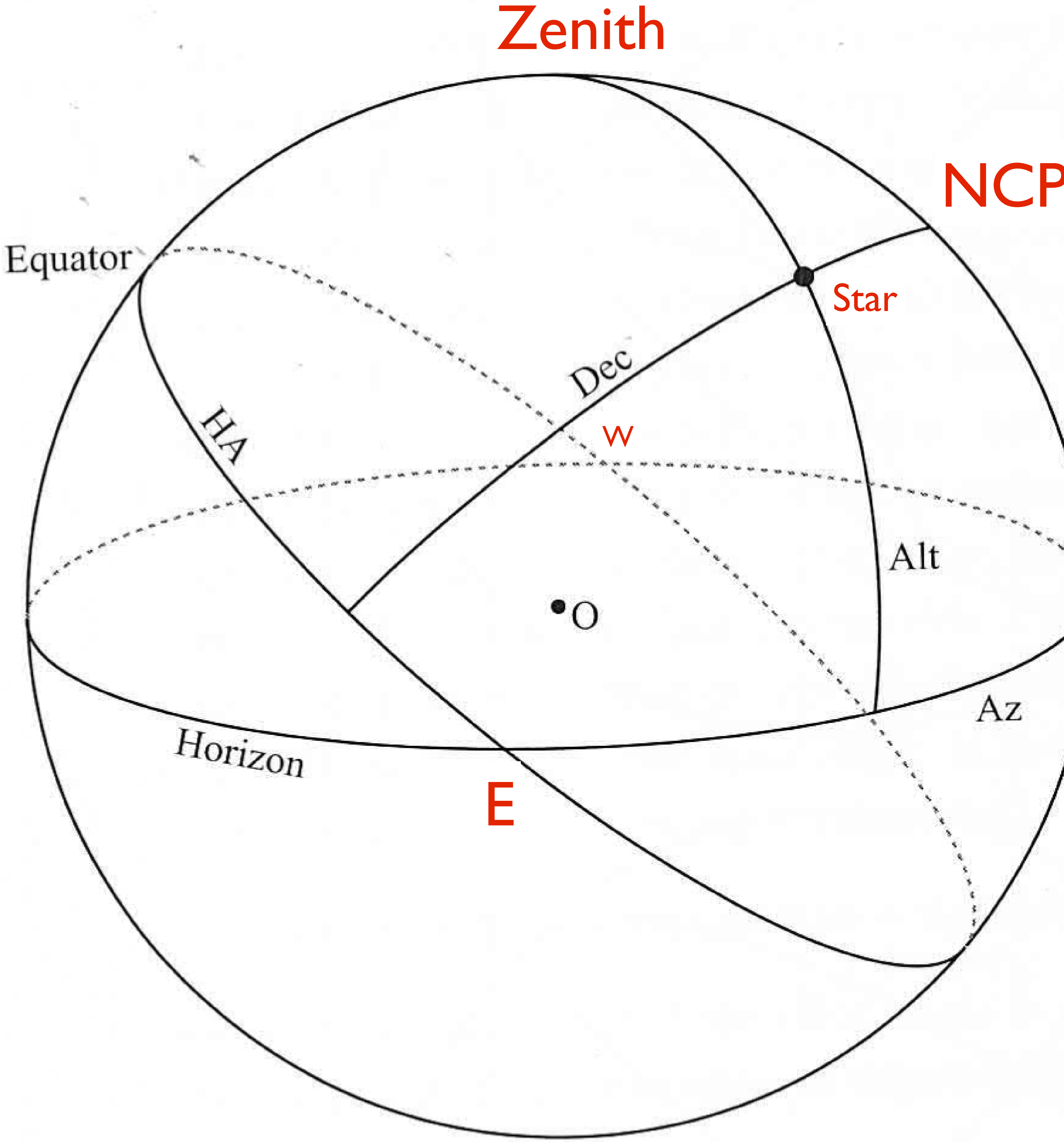
Converting from equatorial to alt az

So, we really need to use -HA.

$$\sin(Az)=\cos(\delta)\sin(-HA)/\cos(alt)$$

But it's worse than that. The arcsine can only recover angles from -90° to +90° unambiguously.




Zenith
NCP
Equator
Star
Dec
HA
W
Alt
O
Az
Horizon
E

Converting from equatorial to alt az

$\sin(Az)=\cos(\delta)\sin(-HA)/\cos(alt)=$

$\cos(41)\sin(-20)/\cos(73.3)= -0.898$

$Az=\sin^{-1}(-0.898)=-63.9^{o}=296.1^{o}$

But, the arcsine is double-valued: sin(x)=sin(180-x). So it could be 243.9^{o}. How can you tell?

Review question

You go outside at NAU at local midnight on August 21. There's a star due east, about 70° above the horizon. What's its RA and DEC?

Conversion of coordinates

Should be able to derive the relationship between equatorial coordinates and galactic coordinates!

Good question might be: what is the galactic longitude (l) and latitude (b) for M31 (α=00:40, δ=+41°).

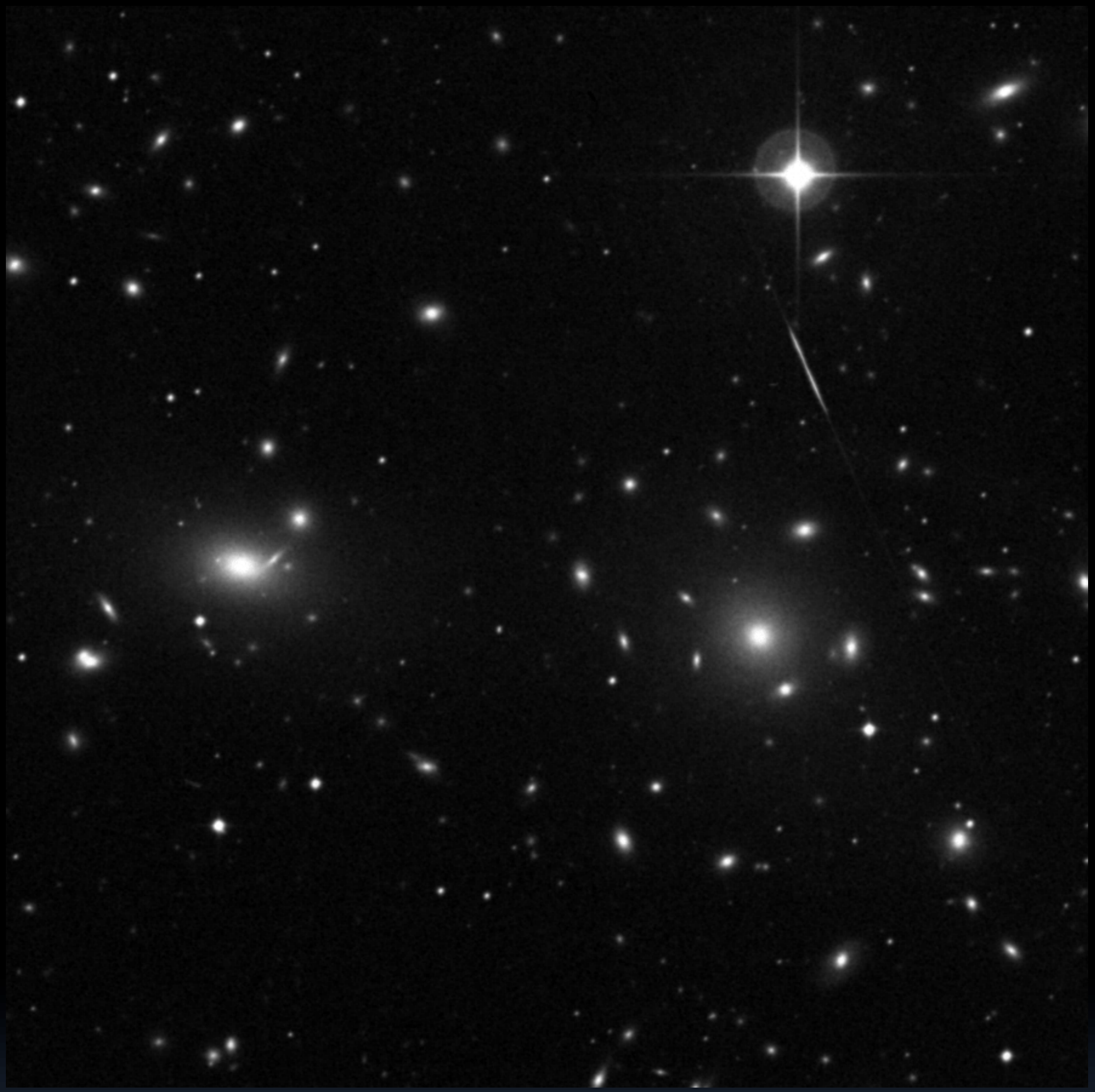
Why would you care?

Low Galactic Latitude

$18^h\ 00^m\ 00^s$ -23° 00′ 00″ (J2000)

l=7° b=0°

High Galactic Latitude



$12^h\ 59^m\ 49^s$ 27° 58′ 50″ (J2000)

l=58° b=88°