Rotating Solar Analogs: How Close to Solar is Good Enough?

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Abstract

Stellar rotational velocity is a poorly-explored parameter contributing to the identification of a star as a solar analog. Rotation contributes directly to stellar activity through the strength and period of the dynamo, and through the creation of "preferred latitudes" of surface activity. It is thus critical to understand the range of permissible rotation period over which the term "solar analog" can be accurately applied.

1 Dynamo Basics

Consider a Parker (1955) migratory dynamo operating in a differentially rotating shell. For large dynamo numbers, it can be described as follows (see, e.g. Kuzanyan & Sokoloff 1997, Parker 1993, Baliunas *et al.* 1996):

$$\frac{\partial A}{\partial t} = \alpha(\theta)B + \frac{\partial^2 A}{\partial \theta^2} \tag{1}$$

$$\frac{\partial B}{\partial t} = -D\cos\theta\frac{\partial A}{\partial \theta} + \frac{\partial^2 B}{\partial \theta^2} \tag{2}$$

where B and A represent the toroidal components of the mean magnetic field and magnetic potential, respectively, D is the dynamo number, α the mean helicity, and θ latitude measured from the stellar equator. In this simplified presentation, I neglect any radial gradient of the stellar rotation.

As is well known, this system of equations has solutions in the form of a harmonic wave:

$$A = |D|^{-\frac{2}{3}}a(\xi)B_0 \tag{3}$$

$$B = b(\xi)B_0 \tag{4}$$

where $a = a_* e^{i\xi}$, $b = b_* e^{i\xi}$, and $\xi = (\omega - i\gamma)t = k\theta$.

1.1 Dynamo Solutions

Solution of these equations (Kuzanyan & Sokolov 1996) begins with choosing the correct scaling, which permits the removal of the dynamo number D from the equations. Such a scaling is:

$$\gamma = |D|^{\frac{2}{3}}\Gamma \tag{5}$$

$$\omega = |D|^{\frac{2}{3}}\Omega \tag{6}$$

$$k = |D|^{\frac{1}{3}}K \tag{7}$$

In an observational context, these parameters essentially describe the growth rate, period, and scale of the dynamo.

1.2 Dynamo Number

The dynamo number for an $\alpha\Omega$-type dynamo in a convective shell of thickness h and radius R is

$$D = \frac{\alpha R^3}{\beta r}\frac{\partial \Omega}{\partial r} \tag{8}$$

where α is the helicity ($\alpha \approx \frac{l^2\Omega}{H}$, where H is a scale height and l is the turbulent scale length) and β the magnetic diffusivity ($\beta \approx lv$, where v is the turbulent velocity). Consulting detailed stellar interiors models which incorporate convective overshoot (Vandenberg 1997) shows $\frac{l}{Hv}$ to be a slowly varying function of mass for stars in the 0.7 -1 solar mass range. In this case, approximating $\frac{\partial \Omega}{\partial r}$ by $\frac{\Omega}{r}$ and substituting gives $D \approx \frac{l}{Hv}R^2\Omega^2$, or $D \sim R^2\Omega^2$ over the lower main sequence.

2 Exploring Parameter Space

By definition, a solar analog should be as similar to the Sun as possible. However, in practice, a large number of parameters are available for use in comparing stars to the Sun, and vigorous disagreement exists over which parameters are most appropriate (see, *e.g.* Buzasi 1997b). The three parameters which I will explore here are

1. Surface Activity Level (*e.g.* levels of Ca II H & K emission, X-ray emission, etc., should be close to the solar norm)
2. Activity Cycle Period (*e.g.* this must approximately replicate the solar case, with some allowance for variations in the solar cycle length)
3. Active Latitudes (a solar analog should display activity in a latitude range similar to that of the Sun)

Of these parameters, the first two are readily observeable, albeit at some expense in telescope time. The third, however, is not realistically an observable parameter for prospective solar analogs, since techniques for determining active latitudes (and longitudes), such as Doppler Imaging (see, *e.g.* Vogt, Penrod, & Hatzes 1987) are not useful at the relatively slow rotation rates expected for these stars.

3 Surface Activity Levels

Consider an idealized statistical flux tube dynamo as described by Schussler (1980). In such a model, surface magnetic structure and behavior (*i.e.*, the butterfly diagram) is caused by flux tubes rising to the surface from the region of generation at the base of the convection zone. Schussler defines a critical field strength B_C, which essentially enforces equipartition with the local convective energy density. (Numerically, such fields are in the kilogauss range on the lower main sequence; for a more detailed discussion see Buzasi 1997a) Once a specific flux tube reaches such a field strength, it has a finite probability of rising to the surface, a probability which increases with both flux tube age and magnetic field. Schussler writes the eruption probability as

$$P = \frac{Bt - C_1}{C_2 - C_1} \tag{9}$$

where C_1 and C_2 are constants. In a Parker dynamo, $B \sim e^{\gamma t}$ (where I consider only the real part of the complex growth rate γ). For small growth rates (or slow rotation: certainly the case for a solar analog!) I can therefore write

$$P = \frac{1}{C_2 - C_1}[Bt + Bt\gamma - C_1] \tag{10}$$

and for $Bt \approx C_1$ (i.e., in the early stages of the growth of the fluxtubes), $P \sim \gamma$.

Observationally, we are concerned with surface activity, so the relevant parameter is the number (and filling factor) of emerging flux tubes. This depends directly on P, so observationally the strength of the dynamo early on in an activity cycle will vary approximately as $\gamma \sim |D|^{\frac{2}{3}}$.

3.1 Ramifications for Solar Analogs, Part 1

Historically, of course, surface activity levels (as expressed, say, by Zurich sunspot number) vary considerably with the solar cycle. However, the peak numbers for a group of cycles vary significantly less. Examination of spot records since 1840 shows a mean peak number of about 105, with a 1σ variation of about 37, or some 35%. I can therefore adopt as a guiding rule that a "good" solar analog will have a value of γ within 35% of the solar value. Substituting for dynamo number and rewriting γ gives

$$\gamma \sim R^{\frac{4}{3}}\Omega^{\frac{4}{3}} \tag{11}$$

or, differentiating

$$\frac{d\gamma}{\gamma} \approx 0.35 = \frac{4}{3}\frac{d\Omega}{\Omega} \tag{12}$$

In the case of a truly solar-like G2V star, this implies that any value of Ω within about 26% of the solar value would lead to a well-behaved "solar analog". This corresponds to a rotational period between about 19 and 33 days (taking the solar period to be 27 days).

In the case of a lower-mass star, we must first consider the increased rotation rate required to yield the same level of activity as seen in the Sun. For example (to choose an extreme case), a $0.7M_{\odot}$ star (about spectral type K5V), with a radius only $0.63R_{\odot}$, must rotate with a period of 17 days in order to have a similar value of γ to the Sun's. In this case, we get in trouble if the K5V star rotates with a period of much less than about 12.5 days.

4 Dynamo Period

By inspection of the dimensionless scalings shown above, the period of the dynamo varies as $P \sim |D|^{-\frac{2}{3}}$. It is worth noting that fully nonlinear dynamos calculated by various authors give varying results for the power in this relation, ranging from -0.33 (Charbonneau & MacGregor 1997) to -0.61 (Jennings & Weiss 1991). Fortunately, as shown below, adopting the $P \sim |D|^{-\frac{2}{3}}$ rule suggested by the simple dynamo outlined above leads to a more severe constraint than that resulting from a more sophisticated dynamo, so adopting it is not likely to cause problems.

4.1 Ramifications for Solar Analogs, Part 2

As Baliunas *et al.* (1996) note, the solar cycle length since the 18th century has varied from 9 to 13 years, or on the order of 20 - 25%. Thus, let us require as a condition for "good" solar "analogousness" a dynamo period within 25% of the solar value. Following a rationale similar to that outlined in equations (11) and (12) leads to the conclusion that

$$\frac{d\Omega}{\Omega} \approx 0.15 \tag{13}$$

Note that the Charbonneau & MacGregor (1997) and Jennings & Weiss (1991) power- law relations lead to more relaxed constraints on $\frac{d\Omega}{\Omega}$. Thus, the Sun could rotate as rapidly as every 23 days and remain a good solar analog (sic!), while the K5V star discussed above could rotate as rapidly as 14.5 days and remain so. Note that, if one is concerned with the impact of rotation on "solar analogousness," this is a more severe constraint than surface activity levels.

5 Active Latitudes

To determine the "width" of the dynamo wave in latitude, follow Kuzanyan & Sokoloff (1997) in approximating the amplitude of the magnetic field strength as a function of latitude with a parabola, and further adopting the expression

$$\Delta = \sqrt{\frac{-2F}{\frac{\partial^2 F}{\partial \theta^2}}} \tag{14}$$

for the half-width of the parabola described by the function $F(\theta)$. This leads directly to the expression

$$\Delta \sim |D|^{-\frac{1}{3}} \tag{15}$$

for the "width" of the active latitude band *at the region of magnetic flux generation.* Typically, calculations lead to active latitude widths in the tens of degrees at the base of the convection zone.

5.1 Active Latitudes and the Coriolis Effect

Observationally, of course, our interest lies at the stellar surface, and as Buzasi (1997a) and others (Schussler & Solanki 1992, Choudhuri & Gilman 1987) have shown, typically rising flux tubes are deflected by the Coriolis force and emerge at latitudes larger than those at which they are generated.

This issue becomes particularly important for lower-mass stars with deeper convection zones. As Buzasi (1997a) has shown, for such stars even relatively slow rotation is sufficient to cause large deviations from radial paths for rising flux tubes. The key is the relative strengths of the Coriolis and buoyancy forces:

$$\frac{F_C}{F_B} \approx \left(\frac{[R - R_C]P_e}{g} \right)^{\frac{1}{2}} \frac{8\Omega\pi^{\frac{1}{2}}}{B} \tag{16}$$

which is rather complex. Fortunately, fitting a power-law to the results of numerical modeling gives

$$\frac{F_C}{F_B} \sim \frac{\Omega}{R^n} \tag{17}$$

where n is slightly mass-dependent but lies between 2.5 and 3.

The important fact here is that the Coriolis force will change the surface active latitudes for a rapidly-enough rotating star. In particular, since the most extreme deviation possible from radial rise for a flux tube is a path that lies parallel to the stellar rotation axis, the result of rotation will be to both increase the latitudes of surface activity (compared to dynamo activity) and to narrow the active latitude band (since flux tubes starting near the equator are affected more by the Coriolis effect). *For stars with masses less than that of the Sun, this effect dominates the* $\Delta \sim |D|^{-\frac{1}{3}}$ *rule derived from dynamo considerations.*

5.2 Ramifications for Solar Analogs, Part 3

Consider a nominally "solar-like" star with a dynamo generating significant magnetic activity at the base of the convection zone in the latitude range $(5°, 40°)$. Such a dynamo is consistent with both analytic (*e.g.*, Parker 1993, Kuzanyan & Sokoloff 1997) and numerical solutions (MacGregor & Charbonneau 1997), and with solar observations. As viewed from the equator of the star, such an activity belt spans some 56% of the stellar surface area. However, as the star rotates more and more rapidly, the belt rises to higher latitudes and narrows. At 3.5 times the solar rotation rate (a period of a little over a week), the belt spans the latitude range $(30°, 51°)$ at the stellar surface, and its projected area has halved. I will adopt this "50% rule" as my criterion for a good solar analog, that is, when this activity belt occupies only half the projected area (viewed from an inclination angle of 90°) that it would on a slow-rotating Sun, a star ceases to be acceptable as a solar analog. In practice, the "turn-on" of the Coriolis effect as an important factor is so rapid as a function of rotation rate that results are insensitive to the precise criterion adopted.

Note that this criterion is highly mass-sensitive. The K5V star discussed above reaches this cutoff at a rotational period of about 23.5 days, and so all such stars with rotational periods faster than this are poor solar analogs.

6 Conclusions

Results are summarized in Figure 1. For stars hotter than about G6, the critical parameter for "good" solar analogs is the cycle period, while in the cooler stars the latitudinal dependence of surface activity becomes an increasingly important factor. Note that **for stars cooler than about G9, there are no acceptable**

solar analogs. The rotational velocities corresponding to the shortest allowable periods are 2.2 km/s for **all** G and early K stars.

Of course, these results can be easily generalized in the case of more exacting solar analog requirements. For example, demanding that a star behave "like the Sun" in all respects to within 1% leads, by examination of equations (12), (13), and (17), to the rather exacting requirement that the star be within one part in 250 of the solar mass, and rotating with a period within about 4 hours of the solar rotational period. These requirements are probably observationally absurd. In addition, as noted elsewhere (Buzasi 1997b), the Sun itself varies sufficiently (both periodically and stochastically) to make such requirements meaningless. The search for "solar twins" may thus be ill-conceived from the start.

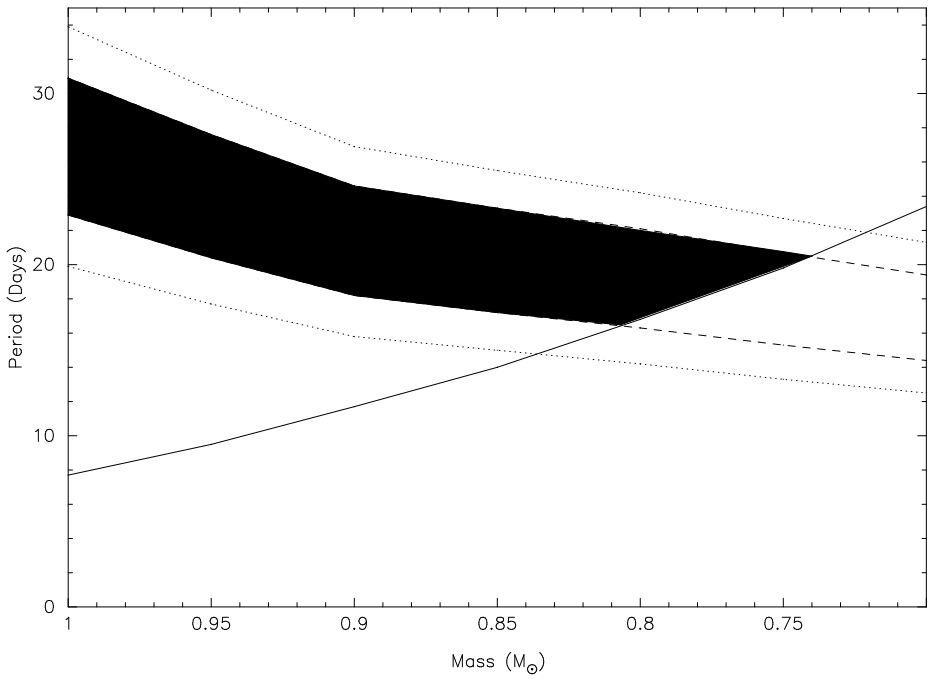

Figure 1: Dotted lines indicate the envelope of acceptable values for rotation as a function of mass under the constraint imposed by requiring surface activity levels to be comparable to that of the Sun, while dashed lines indicate a similar envelope constrained by dynamo period. The solid line marks the dividing line beyond which stellar surface activity (in the form of active latitudes) is dominated by the Coriolis effect. The result of all of these constraints is that only stars lying within the filled region are acceptable solar analogs: this corresponds to stars hotter than spectral type K0.

References

Baliunas, S. L., Nesme-Rimes, E., Sokoloff, D., & Soon, W. H. 1996, ApJ 460, 848.
Buzasi, D. L. 1997a, ApJ 484, 855.
Buzasi, D.L. 1997b, these proceedings.
Charbonneau, P. & MacGregor, K. B. 1997, ApJ 486, 502.
Choudhuri, A. R. & Gilman, P. A. 1987, 316, 788.
Jennings, R. L. & Weiss, N. O. 1991, MNRAS 252, 249.

Kuzanyan, K. M. & Sokolov, D. D. 1996, Astronomy Reports 40, 424.
Kuzanyan, K. M. & Sokoloff, D. 1997, Solar Physics 173, 1.
Parker, E. N. 1955, ApJ 122, 293.
Parker, E. N. 1993. ApJ 408, 707.
Schussler, M. 1980, Nature 288, 150.
Schussler, M. & Solanki, S. K. 1992, A&A 264, L13.
Vandenberg, D. A. 1997, private communication.
Vogt, S., Penrod, G. D., & Hatzes, A. P 1987, ApJ 321, 496.