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ANGULAR DIAMETERS AND EFFECTIVE TEMPERATURES OF CARBON STARS

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ABSTRACT

We report new interferometric angular diameter observations of 15 carbon stars. Combining these with previously published diameters for seven other carbon stars, we are able to compute effective temperatures for 22 stars. Of these, 16 have Yamashita spectral classes and are of sufficiently high quality that we may determine the dependence of temperature on spectral type. We find that (with three exceptions) there is a small dispersion of effective temperatures, in the range of spectral classes C5–C9, with a mean value of 3000±200 K (rms). There is a slight tendency for the effective temperature to increase with increasing temperature index, in agreement with a previous finding of Tsuji. We assess the effects of circumstellar shells upon our temperature determinations and find that only S Aur and CIT 13 are likely to be contaminated by the presence of a shell. The mean radius for the carbon star sample is estimated to be about 400 $R_\odot$, which makes these stars more similar (in effective temperature and radius) to Mira variables than to late M giants. © *1996 American Astronomical Society.*

1. INTRODUCTION

From a statistical analysis of the distribution and mass-loss properties of carbon stars, Claussen *et al.* (1987) inferred that these stars probably originated from F stars in the mass range 1.2 to 1.6 $M_\odot$. Early observations by Mendoza V. & Johnson (1965) and Richer (1971) indicated that this group of stars occupied a region of the H-R diagram populated by the K and M giants. Yet very little fundamental information exists for the carbon star group. Only nine stars were listed in the White & Feierman (1987) catalog, having previously published lunar occultation angular diameters. Quirrenbach *et al.* (1994) have measured the angular size of UU Aur, Y CVn, and TX Psc using the Mark III interferometer and, using additional published lunar occultation diameters and photometry, estimated the effective temperature for nine carbon stars. From their own measured spectrophotometry and published angular diameters, Lázaro *et al.* (1994) have estimated the effective temperatures for four stars.

The effective temperature scale is not very well established owing to these small numbers. In Mendoza V. & Johnson (1965), the temperature estimates range between 2270 and 5500 K. Cohen (1979) adopted a temperature scale, as a function of Yamashita (1972, 1975) spectral class, which ranged from 3240 K at C2 to 2230 K at C9. Quirrenbach *et al.* (1994) compared their effective temperature scale with the scale set by Tsuji (1981a) based upon matching observed fluxes and colors to model atmospheres (the infrared flux method) and found that temperatures determined from angular diameter measurements are about 100 K cooler than those determined from model atmospheres. Goebel *et al.* (1993) have argued that differences of a few hundred K for the effective temperature of TX Psc will determine whether or not the chemical composition is consistent with the existence of a deep envelope. Certainly the atomic line and molecular band strengths in the complex spectra are highly dependent upon the star's surface temperature (Tsuji 1981b). Thus, more angular diameter determinations will help to improve our understanding of the atmospheric properties of this class of stars. Effective temperatures are also an important parameter for understanding the flux spectrum from circumstellar shells surrounding these stars. Rowan-Robinson & Harris (1983) show that the model spectra seem to indicate that the atmospheric temperatures for the earlier types are around 2500 K and for the later types around 2000 K.

It is the purpose of this paper to report new angular diameter measurements of 15 carbon stars made with the IOTA (Infrared Optical Telescope Array) interferometer. Two of these, TX Psc and Y CVn, are common with lunar occultation and Mark III interferometry while 13 are measurements of new stars. We report the first observation of CIT 13, one of the original "infrared stars" (Ulrich *et al.* 1966). We find that there are significant differences in the estimated reddening that, depending upon the method of estimation, will limit the accuracy of the effective temperatures determined from the measured diameters. When we combine our best observations with the sample analyzed by Quirrenbach *et al.* (1994), we obtain effective temperatures for a sample of 16 stars ranging in Yamashita spectral types from C5 to C9. We

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obtain the result that the temperatures increase slightly over this range of spectral types, counter to the original hopes for the spectroscopic classification schemes. This result confirms a previous conclusion by Tsuji (1981a) based upon the infrared flux method.

2. OBSERVATIONS

The data reported here were obtained at IOTA between April and October 1995. Two telescopes on baselines of 21 and 38 meters and a *K* filter ($\lambda=2.2$ μm, $\Delta\lambda=0.4$ μm) were used for all observations. The interferometer and details of the data-taking and reduction process have been described more fully by Carleton *et al.* (1994) and Dyck *et al.* (1995). Briefly, the incoming wave front is divided by the two telescopes and propagated through the interferometer as an afocal beam. Optical delay differences are compensated by fixed and variable delays and the beams are recombined onto a beamsplitter which produces two complementary interference signals. These are monitored by two independent, single-element InSb detectors. In our mode of operation, we drive the variable delay line past the zero-path position at a rate which produces an interferogram with a fringe frequency of 100 Hz. The amplitude of the interferogram at the zero-path position for partially resolved circular disks is related to the source fringe visibility amplitude, with lower amplitudes corresponding to larger diameter sources. The response of the interferometer plus atmosphere is calibrated by observation of stars known to be unresolved and observations of calibrators are alternated with observations of program sources to minimize the effects of variations of the interferometer and atmosphere. The observed fringe visibility of the source divided by the average fringe visibility of calibrators measured before and after the source results in the normalized fringe visibility for the source. The data for the two independent detector channels have then been averaged. These averaged, normalized visibilities are listed in Table 1, along with their standard deviations, the date of the observation, the number of interferograms obtained and the interferometer baseline.

Our observing strategy for measuring stellar disk diameters is to measure one visibility at one telescope separation, corresponding to a single spatial frequency. Neglecting the effects of surface structure and limb darkening, a single spatial frequency will allow a unique determination of the angular diameter. The procedure involves finding the best-fitting uniformly bright, circular disk visibility function (the Bessel function J_1) to the observed datum to yield the uniform-disk angular diameter, θ_{UD}. For a source of unknown geometry, more than one spatial frequency point is required to understand that geometry. However, for stellar disks, a single visibility measurement in the range $0.15 \leqslant V \leqslant 0.80$ should provide sufficient accuracy for the project described in this paper.

If a source was observed on more than one night, the visibilities and base lines were averaged when the base lines differed by no more than 10%. For Y CVn and TX Psc, observations were made at base lines which differed by more than 10%. Uniform disk diameters were computed for each base line and the final value for each star was the weighted mean of the independent determinations.

TABLE 1. The observed data.

Star	UT Date	B (m)	N	$V \pm \varepsilon_V$
AQ And	95 Oct 08	38.26	100	0.863±0.099
VX And	95 Oct 05	36.90	100	0.681±0.058
V Aql	95 Jul 12	31.13	98	0.511±0.017
S Aur	95 Oct 05	37.15	34	0.478±0.053
WZ Cas	95 Oct 04	34.73	100	0.777±0.051
Y CVn	95 Apr 28	20.50	159	0.702±0.052
Y CVn	95 Apr 29	20.58	158	0.643±0.013
Y CVn	95 May 01	20.60	196	0.655±0.009
Y CVn	95 Jun 03	37.71	25	0.251±0.009
Y CVn	95 Jun 04	37.69	169	0.220±0.007
Y CVn	95 Jun 05	37.68	398	0.200±0.004
RS Cyg	95 Oct 08	38.22	98	0.849±0.130
RV Cyg	95 Oct 08	38.24	100	0.577±0.084
V460 Cyg	95 Jul 10	37.08	76	0.704±0.051
W Ori	95 Oct 07	32.94	99	0.501±0.057
RT Ori	95 Oct 07	34.46	100	0.866±0.130
SY Per	95 Oct 04	34.98	100	0.895±0.009
SY Per	95 Oct 05	36.41	50	0.938±0.158
Z Psc	95 Oct 08	38.21	97	0.810±0.123
TX Psc	95 Jul 08	36.90	28	0.320±0.029
TX Psc	95 Oct 06	33.20	50	0.514±0.122
TX Psc	95 Oct 07	34.09	50	0.478±0.091
CIT 13	95 Jul 09	37.59	269	0.252±0.005
CIT 13	95 Jul 10	36.95	56	0.336±0.013

We feel that 2.2 μm is a good choice of wavelengths for the observations because (1) it lies near, but on the Rayleigh-Jeans side of, the flux spectrum peak wavelength and (2) the effects of circumstellar dust on the determination of the atmospheric angular diameter will be minimized. The second assumption is based upon the following arguments, first put forward by Tsuji (1978): Small circumstellar dust particles generated by the star will absorb, scatter, and thermalize the photospheric radiation. Owing to their small size, the absorption and scattering efficiencies decrease with increasing wavelength. Hence, longer wavelengths are better than shorter ones from this point of view. However, as the wavelength increases, the thermal re-emission spectrum of the grains becomes increasingly more important. If the grain temperatures are less than about 1000 K (Rowan-Robinson & Harris 1983), then the peak of the re-emission spectrum will be at wavelengths longer than about 3 μm, with shorter wavelengths lying on the steeply declining exponential part of the spectrum. Hence, 2.2 μm seems to be an ideal wavelength.

However, all of the stars in our sample have measurable mass loss, with rates $d\mathbf{M}/dt$ greater than about 10^{-7} $\mathbf{M}_\odot$ yr $^{-1}$ (Claussen *et al.* 1987). This implies the existence of a circumstellar shell which may affect our determination of the effective temperatures. Judging from the models of Rowan-Robinson & Harris (1983), we believe that most of the stars in our sample (as we shall see later, there are two or three important exceptions) will have 2.2 μm optical depths much less than 0.1. We will discuss the possible effects of shells on our angular diameters in a later section.

The sample of stars selected depended upon brightness, availability, and variability class. We have restricted the sample discussed in this paper to semi-regular and irregular variable stars. Mira variables have been specifically excluded from the discussion, although we have observed a number of these, and they will be the subject of a future paper. Most of the observations reported here were made on a single night, with 50–100 interferograms typically obtained for an observation. Our previous experience (Dyck *et al.* 1996) has shown us that this is sufficient in a statistical sense to characterize the angular diameter of bright stars with visibilities near 50%. The errors listed for the angular diameters, however, were based upon the rms fluctuations in the visibility of stars observed repeatedly over many nights. We have found that this error better characterizes the uncertainty in the visibility and, hence, the angular diameter than does the standard deviation listed in Table 1. The formula for assigning this "external" error to the visibilities is

$$\epsilon_v = \pm \frac{0.051}{\sqrt{\text{number of nights}}}.$$

The interested reader should see Dyck *et al.* (1996) for a more complete description.

As we have previously noted (Dyck *et al.* 1996), there appears to be a systematic difference between the uniform-disk diameters determined at IOTA compared to those determined at the same wavelength with the I2T interferometer at CERGA. The difference is in the sense that stars with diameters in the range of 10–20 milliarcsec (mas) are about 10% smaller measured with IOTA than with CERGA. For stars with smaller diameters, there appears to be no measurable difference. A 10% difference in the angular diameter corresponds to a 2.5% difference in the effective temperature scales for the two interferometers.

3. EFFECTIVE TEMPERATURES

To compute the effective temperatures we need the bolometric flux, $F_{\rm bol}$, corrected for reddening, and the angular diameter which corresponds to the average radiating surface for the star, θ_R. Then, we use the relationship

$$T_{\rm eff} = 1.316 \times 10^7 \left(\frac{F_{\rm bol}}{\theta_R^2} \right)^{1/4} \ {\rm K},$$

where $F_{\rm bol}$ is measured in W cm^{-2} and θ_R is measured in mas.

Previous estimates of the effective temperature and luminosity for carbon stars (Mendoza V. & Johnson 1965; Richer 1971) have indicated that these stars have properties close to the K and M giants. Hence, atmospheric extensions ought to play a relatively small role in the determination of the effects of limb darkening. As before, we have used the model atmospheres of Scholz & Takeda (1987) to evaluate limb-darkening effects, and have adopted the surface where the Rosseland mean opacity is unity as the appropriate surface for computing an effective temperature. The conversion from uniform disk angular diameters to Rosseland mean angular diameters is a small multiplicative factor with small dispersion for the models computed by Scholz & Takeda (1987). Following Dyck *et al.* (1996), we have adopted

$$\theta_R = 1.022\theta_{\rm UD}.$$

Tsuji (1981a), on the other hand, has suggested that the carbon stars have temperatures that are lower and luminosities that are higher than the normal M giant sequence, so that some atmospheric extension may be important. From our investigation of the Scholz & Takeda (1987) models, we estimate that the limb-darkening corrections could be no more than 10% different from our correction factors (i.e., the correction from uniform-disk diameters to Rosseland mean diameters). This could lead to a systematic error of 5% in the effective temperature scale; we shall return to this point.

The Rosseland mean opacity surface has the advantage that the effective temperatures will not be dependent upon the wavelength of the observation. For other limb-darkening corrections, the temperatures computed will be corrected for center-of-limb brightness variations but will still be a function of wavelength since the brightness profiles at each wavelength will, in general, correspond to different altitudes in the atmosphere. The different altitudes will lead to a wavelength-dependent limb-darkened angular diameter and, hence, an effective temperature which is a function of wavelength.

To compute the bolometric flux densities, we have made extensive use of the data compiled by Gezari *et al.* (1993) for wavelengths longer than 1 μm. Shorter wavelength data were obtained from Mendoza V. & Johnson (1965), Johnson *et al.* (1966), Eggen (1967), and Richer (1971). We converted magnitudes from filters *B* through *L* into absolute fluxes and performed a simple numerical integration over this wavelength range. Fluxes beyond 3.5 μm were estimated by taking a Rayleigh-Jeans integration, normalized to the flux in the *L* passband. Variability is noted for these carbon stars but, from an inspection of the average infrared variation for our stars given in Gezari *et al.* (1993), we believe that the bolometric flux generally changes by no more than 10%. In cases where photometry is missing, we have used mean colors for the spectral type, tied to the observed *K* magnitude. For the purposes of estimating the errors from the bolometric flux, we have assumed a conservative 15% for variability and errors in the absolute calibration, unless significant photometry was not available. For this case, we have explored the extremes of the colors versus spectral types and computed corresponding extreme values of the bolometric flux. Differences in the effective temperatures obtained from these extreme flux values were added in quadrature to other sources of error to obtain the final estimates of the error in the temperature.

Reddening may also be important and, in some cases, different estimates thereof vary wildly. We have compared the reddening determinations made by Richer (1971) at optical wavelengths with those made by Claussen *et al.* (1987) at near-infrared wavelengths. Each determination rests upon

TABLE 2. Effective temperatures from the IOTA interferometry.

Star	F_{bol} (W cm^{-2})	$\theta_{UD} \pm \varepsilon_\theta$ (mas)	θ_R (mas)	$T_{EFF} \pm \varepsilon_T$ (K)	Sp Type	dM/dt (M⊙ yr^{-1})	Error Source
VX And	1.87×10^{-13}	6.6±0.6	6.75	3332±598	C4,5	1.2×10^{-7}	R
AQ And	7.12×10^{-14}	4.0±0.8	4.09	3362±394	C5,4	2.5×10^{-7}	R=D
V Aql	4.36×10^{-13}	10.1±0.7	10.32	3328±480	C5,4	1.3×10^{-7}	R
S Aur	6.36×10^{-14}	8.9±0.6	9.10	2191±219	---	6.8×10^{-7}	F
WZ Cas	1.14×10^{-13}	5.8±0.7	5.93	3140±193	C9,2	6.8×10^{-8}	D
Y CVn	3.62×10^{-13}	11.6±0.3	11.86	2964±52	C5,5	1.1×10^{-7}	R=D=F
RS Cyg	7.84×10^{-14}	4.3±0.8	4.39	3321±312	C8,2	1.0×10^{-7}	D
RV Cyg	1.21×10^{-13}	7.6±0.5	7.77	2784±97	C6,4	2.2×10^{-7}	D
V460 Cyg	1.45×10^{-13}	6.3±0.6	6.44	3200±157	C6,3	1.6×10^{-7}	D
W Ori	3.34×10^{-13}	9.7±0.6	9.91	3177±306	C5,4	1.3×10^{-7}	R
RT Ori	4.38×10^{-14}	4.4±0.9	4.50	2839±292	C6,4	1.4×10^{-7}	D
SY Per	3.57×10^{-14}	3.4±0.8	3.47	3069±363	C6,4	8.9×10^{-8}	D
Z Psc	8.84×10^{-14}	4.8±0.7	4.91	3240±239	C7,2	9.3×10^{-8}	D
TX Psc	3.18×10^{-13}	11.2±0.3	11.44	2921±60	C7,2	7.2×10^{-8}	D=F
CIT 13	3.39×10^{-14}	10.8±0.4	11.04	1700±37	---	2.8×10^{-6}	?

a different set of assumptions and in many cases the results agree well. For example, for TX Psc, Richer obtains $A_V = 0.25$ while Claussen *et al.* obtain $A_V = 0.22$. On the other hand, reddening estimates for VX And are very discrepant, with Richer (1971) obtaining $A_V = 6.1$ and Claussen *et al.* obtaining $A_V = 0.8$! When two estimates of the reddening were available, we computed values of the bolometric flux using each. Differences in the effective temperatures from these reddening values were added quadratically to the other sources of error to obtain total errors in the effective temperatures.

We have listed our new effective temperature determinations in Table 2, where we have given the name of the star, the Yamashita spectral type, the uniform-disk angular diameter, the error in the measured diameter, the Rosseland mean angular diameter, the adopted bolometric flux, the effective temperature, the error in the effective temperature, the mass-loss rate from Claussen *et al.* (1987), and the major source of the error in the effective temperature, where D=angular diameter, R=reddening, and F=bolometric flux. In addition to our new observations, we have compiled the limb-darkened angular diameters discussed by Quirrenbach *et al.* (1994) in Table 3 and computed their effective temperatures after redetermining the bolometric flux (it is not clear how Quirrenbach *et al.* dealt with the reddening). These stars bring the total number in the sample to 22.

4. DISCUSSION

Four of the stars listed in Tables 2 and 3 have estimated errors in the effective temperature which are larger than about 400 K; these are VX And, AQ And, V Aql, and RT Cap. It is interesting to note that, for these four stars, uncertainties in the interstellar reddening dominate the error in the effective temperature. We have excluded these four from further analysis. We begin our discussion with a comparison of the effective temperatures computed for TX Psc and Y CVn, stars that have angular diameter measurements made with a variety of techniques. It has been noted from repeated measures of TX Psc that there may be a substantial variation in the angular diameter which is correlated with the visual magnitude (Quirrenbach *et al.* 1994). We note that our effective temperature agrees very well with the value obtained from the average Michelson interferometry carried out by Quirrenbach *et al.* but disagrees substantially with the lunar occultation determinations (Lasker *et al.* 1973; de Vegt 1974; Dunham *et al.* 1975). The sense of the disagreement with the lunar occultations is that the IOTA effective temperature estimate is lower than the one made from the occultations. Most of this disagreement appears to be the result of the time variations noted for TX Psc. On the other hand, our observations of Y CVn indicate a smaller angular diameter than does the optical wavelength interferometry of Quirrenbach *et al.* Our effective temperature determination for this star is about 300 K higher than theirs. Thus, we see no consistent pattern in the measurements made in common, but there may be as much as a few hundred K scatter among the effective temperatures determined from lunar occultations, the Mark III optical interferometer and the IOTA (near-infrared) interferometer.

TABLE 3. Supplemental effective temperature data.

Star	F_{bol} (W cm^{-2})	$\theta_{LD} \pm \varepsilon_\theta$ (mas)	$T_{EFF} \pm \varepsilon_T$ (K)	Sp Type	dM/dt ($M_\odot$ yr^{-1})	Error Source
Y Tau	1.42E-13	8.4±1.0[1]	2787±169	C6,4	2.2×10^{-7}	D
UU Aur	3.43E-13	12.1±0.2[2]	2899±82	C6,4	1.3×10^{-7}	R
X Cnc	1.11E-13	8.2±0.6[1]	2653±102	C5,4	1.0×10^{-7}	D
Y CVn	3.62E-13	14.8±0.5[2]	2655±52	C5,5	1.1×10^{-7}	R=D=F
TW Oph	7.59E-14	10.4±0.5[1]	2147±58	C5,5	1.1×10^{-7}	D=F
SZ Sgr	3.05E-14	3.3±0.2[1]	3009±84	C7,3	1.8×10^{-7}	D
AQ Sgr	1.3E-13	6.0±0.5[1]	3234±257	C7,4	1.4×10^{-7}	R
RT Cap	2.47E-13	8.0±0.2[1]	3274±580	C6,4	9.5×10^{-8}	R
TX Psc	3.18E-13	9.3±0.8[1]	3239±136	C7,2	7.2×10^{-8}	D
TX Psc	3.18E-13	11.2±1.0[2]	2953±136	C7,2	7.2×10^{-8}	D

Notes to Table 3: [1] Angular diameter obtained from lunar occultation. [2] Angular diameter obtained from the Mark III interferometer. See Quirrenbach et al. (1994) for details.

There are sixteen stars in the sample with errors in the effective temperature less than about 400 K and that have Yamashita spectral types, intended to be a temperature sequence following suggestions by Keenan & Morgan (1941). Our stars range in spectral class from C5 to C9, where Cohen has suggested effective temperatures ranging from 2750 down to 2230 K, respectively. We have plotted the effective temperatures for these sixteen stars, as a function of spectral type, in Fig. 1. One sees in this figure that the average effective temperature for the group is slightly less than 3000 K and that there is only a weak dependence upon spectral type.

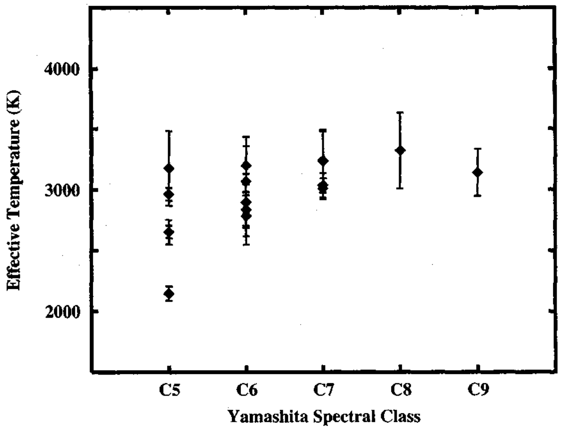

FIG. 1. A plot of the effective temperatures vs Yamashita (1972, 1975) spectral type for the carbon stars discussed in this paper.

TABLE 4. Mean effective temperatures vs Yamashita spectral class.

Spectral Type	$T_{EFF}\pm\varepsilon_T$ (K)
C5	2880±156
C6	2928±69
C7	3130±62
C8	3321±96
C9	3140±96

Furthermore, the dependence is opposite to that desired from the spectroscopic classification, namely, that the temperature actually increases toward the "later" spectral types. This behavior has been previously noted by Tsuji (1981a,b) from effective temperature calculations based on the infrared flux method. Since the Yamashita spectral sequence is based largely upon the strengths of diatomic molecular bands, perhaps the large dispersion in band strengths implies the existence of different levels of mixing, as suggested by Goebel *et al.* (1993), rather than a change in effective temperature.

Our temperature scale is warmer than the one adopted by Cohen (1979) by about 250 K at the early types to about 750 K at the later types; compared to the temperature scale adopted by Rowan-Robinson & Harris (1983) our scale is 500–1000 K warmer. For nine stars in common, our effective temperature scale is about 200 K warmer than Tsuji's (1981a) scale. Note that, in Fig. 1, there is one star which seems to have a much lower temperature than all the others: TW Oph, classified C5, has an effective temperature of about 2150 K. Its angular diameter was determined by lunar occultation methods (Ridgway *et al.* 1982). We will return to this star later in the discussion.

In Table 4, we have summarized the average temperature for each spectral class, along with the estimated dispersion at each spectral type. At spectral class C5, we have excluded TW Oph. Where only one temperature exists, we have taken the rms error for all the multiple observations in the other spectral classes. Note that the averages vary from about 2900 K for type C5 up to about 3150 K at type C9. The mean over all spectral types C5–C9 (excluding TW Oph) is about 3000 ±200 K (rms variation).

There are three stars in Tables 2 and 3 which have significantly lower temperatures than the remainder of the sample. These are S Aur, CIT 13 and TW Oph, of which only TW Oph has a Yamashita classification. It is possible that undetected circumstellar emission or scattering will affect the determination of the effective temperature and it will generally be in the sense of lowering the temperature. The argument is the following: A measurable presence from a circumstellar shell will appear as an angular diameter which is larger than the stellar surface. The increase in size will depend upon the ratio of circumstellar to stellar flux and, hence, on the dust optical depth of the shell. An increase in angular size for a given flux will correspond to a lower temperature.

In order to investigate the possibility that the undetected presence of circumstellar dust has affected the temperature determinations, we have correlated the temperatures in Tables 2 and 3 with the mass loss rates determined by Claussen *et al.* (1987). A higher mass loss rate will generally correspond to a larger shell optical thickness and, hence, to a larger fractional flux observed from shell emission and scattering. Although the mass loss rates determined by these authors are total rates obtained by assuming a constant gas-to-dust ratio, they have been determined from the excess far-infrared flux and correspond essentially to properties of the dust. In Fig. 2 we have plotted the data for all 22 stars observed on this program. There are three important points to note about the figure. First, there is a clustering of stars near effective temperature 3000 K and mass loss rate $dM/dt = 1 \times 10^{-7} M_\odot\ \mathrm{yr}^{-1}$. We propose that this group of stars is unaffected by circumstellar shells. Second, there are two stars (S Aur and CIT 13) with significantly higher estimated mass loss rates than this first group which also have significantly lower effective temperatures. We believe that our temperature estimates have been corrupted by the presence of circumstellar shells for these two stars. Third, TW Oph with a low mass loss rate also has a significantly lower temperature than stars in the first group.

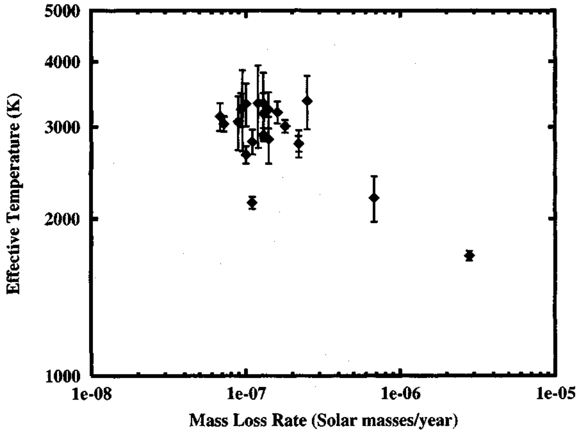

FIG. 2. A plot of the estimated mass loss rates from Claussen *et al.* (1987) vs the effective temperatures. Note that there is a group of stars which cluster at about $T_{\mathrm{eff}} = 3000$ K and $dM/dt = 10^{-7} M_\odot\ \mathrm{yr}^{-1}$. The two stars with the highest mass loss rates are S Aur and CIT 13. The remaining star with very low effective temperature is TW Oph, discussed more fully in the text.

We believe that the low temperature for this star is not affected by the presence of a circumstellar shell. It also appears unlikely that TW Oph represents an isolated example of a rare group of carbon stars with higher luminosity since the circumstellar CO expansion velocity is only a modest 9 km/s (Loup *et al.* 1993; but see the discussion of carbon star luminosities by Zuckerman *et al.* 1986). Perhaps this star is a member of a class of lower temperature carbon stars; this point may be addressed by observation of a much larger sample of carbon stars. On the other hand, it was emphasized to us by the referee that TW Oph lies in the direction of the Galactic Center, where reddening may be very important. Our assumed reddening was $A_V = 0.77$, obtained from Claussen *et al.* (1987). In order to raise the effective temperature of this star to the average of the other C5 stars the visible reddening would have to be of order $A_V \sim 5.7$. This difference, with respect to Claussen *et al.*, is of the same order as noted above for VX And and cannot be ruled out at this time.

One of the important conclusions of this research is that there is a relatively small range of effective temperatures for most of the stars observed in our sample. This temperature, approximately 3000 K in the mean, is about 200 K cooler than the M7 luminosity class III stars recently discussed by Dyck *et al.* (1996). This confirms the result of Tsuji (1981a,b) that the carbon stars were cooler than the earlier M giants.

Claussen *et al.* (1987) assumed that galactic carbon stars have the same narrow range of absolute K magnitudes which are observed for carbon stars in the Magellanic Clouds (Frogel *et al.* 1980) and infer a mean bolometric luminosity of $10^4 L_\odot$. If we take this mean luminosity and couple it with our mean effective temperature, we deduce a mean radius for C5–C9 carbon stars of about 400 $R_\odot$. This may be compared to the mean radius 80 $R_\odot$ determined from parallax data for the M4 giants (Dyck *et al.* 1996). On the other hand, van Belle *et al.* (1996) have shown from statistical distance arguments that the oxygen-rich Mira variables have near-infrared radii averaging 376 $R_\odot$; the effective temperatures for this sample of Miras average 2700 K. Taking radii and effective temperatures together, carbon stars observed in the present program are closer to the Mira variables than to the M giants among the oxygen-rich stars. This finding is consistent with, for example, the evolutionary sequence for carbon stars suggested by Willems (1987). In his picture, the non-Mira carbon stars are the direct descendants of evolving oxygen-rich Mira variables in which pulsations have been largely quenched.

The large radii present a potential problem for our previous assumptions about the conversion from uniform-disk diameters to Rosseland mean diameters. In van Belle *et al.* (1996), it was found that the conversion factor ranged from 0.98 to 1.11 for Mira stars of effective temperature near 3000 K, depending upon whether the star was near maximum or minimum light. If the limb-darkening correction was at either of these extremes it could result in a 5% error in the effective temperature scale, in either direction. Specifically, if the angular diameter correction factor were 1.11, then our scale would be depressed by about 150 K, agreeing more closely with the infrared flux method scale (Tsuji 19881a,b). These differences in temperature scale may become clearer with additional model atmospheres calculations.

5. CONCLUSION

In this paper we have discussed new interferometric determinations of angular diameters for 15 carbon stars. We have combined these data with lunar occultation and Michelson interferometry data for 7 other stars found in the literature. Coupling the angular diameters with bolometric fluxes we have computed effective temperatures for 22 stars. Using the statistical luminosity determinations from Claussen *et al.* (1987), we infer the mean radius for the sample. We find the following:

(1) There is a small range of effective temperatures for stars classified C5–C9 (Yamashita 1972, 1975), with a tendency for later spectral types to be hotter stars. The mean temperature, excluding three peculiar examples is 3000±200 K.

(2) There is generally no evidence for the effects of undetected circumstellar shells upon the effective temperature determinations, except for two stars (S Aur and CIT 13). For these two stars, that have mass loss rates near $10^{-6} \mathbf{M}_\odot$ yr $^{-1}$, the effective temperatures are significantly lower than the ones described in point 1 above.

(3) The statistical mean radius for 20 stars in the sample (S Aur and CIT 13 excluded) is 400 $R_\odot$. This radius makes the carbon stars as extended as the average Mira variable discussed by van Belle *et al.* (1996). A consequence of the extension is that there may be a systematic error of ±150 K resulting from an incorrect choice of limb-darkening correction.

(4) There is one star (TW Oph) which has an anomolously low effective temperature, which may indicate a relatively rare occurrence of lower temperature carbon stars or may simply indicate a very significant error in the reddening estimate.

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