# 1 The Basics of the Exposure Time Calculator

The DCT Exposure Time Calculator (ETC) is based upon the Appendix in the Direct Imaging Manual for Kitt Peak (Massey et al. 1992, 2002)[1]. The signal-to-noise ratio (SNR) is calculated as follows. First, the signal is simply the count rate (e/sec) from the star $N_*$ measured through some aperture times the exposure time $t$. The noise is make up of three components, which are added in quadrature:

- The photon noise from the star itself, $\sqrt{N_* \times t}$.
- The photon noise from the sky, $\sqrt{S \times p \times t}$, where S is the count rate per pixel (e/sec/pixel) and $p$ is the number of pixels in the measuring aperture.
- The read-noise from the device $\sqrt{p} \times R$ in the measuring aperture, where $R$ is the read-noise per pixel.

Put this all together and we have:

$$\mathrm{SNR} = \frac{N_* \times t}{\sqrt{N_* \times t + S \times p \times t + p \times R^2}} \tag{1}$$

In doing this, we have ignored dark current (which is usually negligible), and flat-fielding errors.

## 1.1 Where do the individual parameters come from?

We assume that the number of pixels $p$ is dependent on the seeing, and that the astronomer will always measure the brightness of a star using a near-optimal measuring aperture, i.e., one that has a radius that is about 0.67× the full-width at half-maximum (fwhm). In that case the number of pixels $p$ will be $\pi \times (0.67 \times \mathrm{fwhm})^2 = 1.4 \times \mathrm{fwhm}^2$. If the number of pixels is less than 9, we impose a minimal value of 9, corresponding to a radius of 2.5 pixels.

With each of the telescope/CCD combinations, we have measured the observed count rates (e/sec) and then normalized the values to a $U = B = V = R = I = 20$ mag star observed, correcting for extinction to outside the earth's atmosphere (airmass $X = 0.0$). Let's call these values $N_{20}$. Thus if we want to know $N_*$ for a star of magnitude m that we plan to observe at an airmass of X we compute:

$$m_{\mathrm{corrected}} = m + \mathrm{extinction} \times X$$
$$N_* = N_{20} \times 10^{-(m_{\mathrm{corrected}}-20.)/2.5}$$

The sky value $S$ (in e/sec/pixel) is a little tricker to compute. First, it clearly is a function of lunar phase, particularly at the bluer wavelengths. Massey et al. (1992, 2002) have tabulated the sky brightness (in mag/arcsec[2]) at various lunar phases, and Frank Valdes performed a simple low order fit to these for use in ccdtime, and we adopt these

[1] http://www.noao.edu/kpno/manuals/dim

[2] We leave it as an exercise for the student to determine why $r = 0.67 \times \mathrm{fwhm}$ is essentially the optimal measuring radius.

values here. Yes, the actual sky brightness depends (a lot) on how far away from the moon you are pointing, but we are trying to give reasonable ballpark estimates here. So, we know the sky brightness $s$ in mag/arcsec$^2$. The counts you expect from the sky per arcsec$^2$ is thus $N_{20} \times 10^{-(s-20)/2.5}$. The area of a pixel is the scale$^2\times$ binining$^2$ (where the scale is in arcsec/pixel) and so

$$S = N_{20} \times 10^{-(s-20)/2.5} \times \text{scale}^2 \times \text{binning}^2.$$

The read-noise $R$ is a value you measure in the lab (and at the telescope), and is typically $\sim$4-7 e.

## 1.2 How do we do it?

Given any two parameters, we want the third. Here are the equations we use, given the above.

### 1.2.1 Case 1: Given a magnitude and a SNR, what exposure time do we want?

This is dealt with explicitly in Massey et al. (1992, 2002). If we solve Equation (1) for t, we find ourselves with our old friend the quadratic equation:

$$t = \frac{-B + \sqrt{B^2 - 4 \times A \times C}}{2 \times A}$$

where

$$A = N_*^2$$

$$B = (\text{SNR})^2 \times (N_* + p \times S)$$

$$C = (\text{SNR})^2 \times p \times R^2$$

### 1.2.2 Case 2: Given a magnitude and exposure time, what SNR to we achieve?

This is the easiest case: we just plug our values into Equation (1).

### 1.2.3 Case 3: Given a SNR and exposure time, what magnitude do we reach?

This requires first solving Equation (1) for $N_*$, which again requires the quadratic equation:

$$N_* = \frac{-B + \sqrt{B^2 - 4 \times A \times C}}{2 \times A}$$

where

$$A = t^2$$

$$B = -t \times \text{SNR}^2$$

$$C = -\text{SNR}^2 \times S \times p \times t + p \times R^2$$

Once we have $N_*$ we must turn it into a magnitude:

$$m_{\mathrm{raw}} = -2.5 \times \log N_*/N_{20} + 20$$

That's the brightness of a star outside the earth's atmosphere. But we aren't going to go as faint as that, as we are observing at some finite airmass $X$. So,

$$m_{\mathrm{corrected}} = m_{\mathrm{raw}} - \mathrm{extinction} \times X$$

## 1.3 Output

We give the bandpass, magnitude, S/N, and exposure time, irrespective of which two were input and which one is output. We also include the number of pixels in the measuring aperture, the sky brightness (in e/pixel), the number of e from the star, the peak number of e from the star (see below), and the noise contributions from the three components.

The peak number of counts from the star $P$ will be (roughly):

$$P = N_* \times t/(1.13 \times \mathrm{fwhm}^2),$$

where the fwhm is in pixels.

Table 1. Count Rates Assumed $N_{20}$ e/sec/image

| Tel/Inst | U | B | V | R | I | VR | H$\alpha$-on | H$\alpha$-off | Readnoise R |
|---|---|---|---|---|---|---|---|---|---|
| DCT/LMI | 100 | 730 | 680 | 670 | 450 | 990 | 15 | 70 | 7.0 |
| Hall/NASA42 | 3.1 | 17.1 | 17.5 | 19.7 | 12.8 | 25.0 | — | — | 4.6 |